Solution of non-linear free surface problems

Inspired by variational principles

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This work was carried out within the context of D Manolas' PhD

Outline

1. Problem formulation
   Within the context of potential theory
2. Numerical reformulation
   Integral eq, Free Surface Dynamic eqs, Far-field conditions
3. Numerical results
   examples
4. Outlook

Problem formulation

Wave-current-body interaction

Equations

$$\Phi(\mathbf{x};t)=\varphi(\mathbf{x};t)+U_0\ x\frac{\delta y}{\delta x}$$

$$\nabla^2\varphi=0,\quad in\, D(t)$$

Free – surface conditions on $S_{FS}(t)$ *at* $z=\zeta(x;t)$

Eulerian

$$\frac{\partial\varphi}{\partial t}=-g\ \zeta-\frac{1}{2}\left|\nabla\varphi\right|^2-U_0\frac{\partial\varphi}{\partial x}-\frac{U_0^2}{2}-\frac{p_a}{\rho}-damp_1,$$

$$\frac{\partial\zeta}{\partial t}=-\nabla\varphi\cdot\nabla\zeta-U_0\frac{\partial\zeta}{\partial x}+\frac{\partial\varphi}{\partial z}-damp_2,$$

Lagrangian

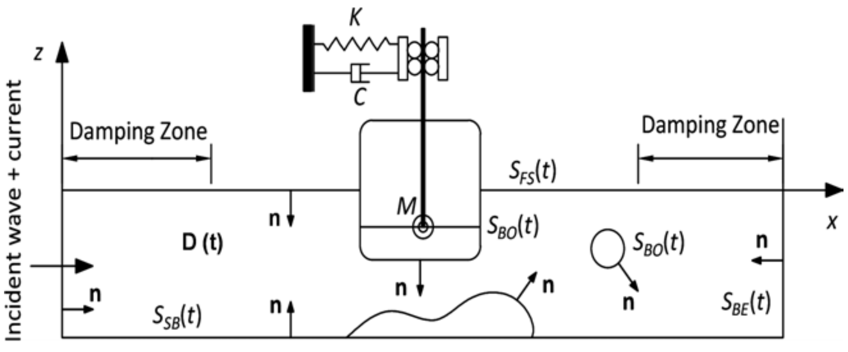
$$\frac{d\varphi}{dt}=-g\ \zeta+\frac{1}{2}\left|\nabla\varphi\right|^2-\frac{U_0^2}{2}-\frac{p_a}{\rho}-damp_1$$

$$\frac{d\mathbf{x}}{dt}=\nabla\varphi+U_0\ \delta_{ix}-damp_2\ \delta_{iz}$$

Semi – Lagrangian

$$\frac{d\varphi}{dt}=-g\ \zeta-\frac{1}{2}\left|\nabla\varphi\right|^2-U_0\frac{\partial\varphi}{\partial x}-\frac{U_0^2}{2}+\frac{d\zeta}{dt}\cdot\frac{\partial\varphi}{\partial z}-\frac{p_a}{\rho}-damp_1,$$

$$\frac{d\zeta}{dt}=-\nabla\varphi\cdot\nabla\zeta-U_0\frac{\partial\zeta}{\partial x}+\frac{\partial\varphi}{\partial z}-damp_2$$



$damp_1=v_1(x)(\partial_n\varphi-\partial_n\varphi^{Wave})$, for the dynamic FS condition

$damp_2=v_2(x)(\zeta-\zeta^{Wave})$, for the kinematic FS condition

Problem formulation

Wave-current-body interaction

Equations

$$\Phi(\mathbf{x};t)=\varphi(\mathbf{x};t)+U_0\ x\frac{\delta y}{\delta x}$$

$$\nabla^2\varphi=0,\quad in\ D(t)$$

$Free-surface\ conditions\ on\ S_{FS}(t)\ at\ z=\zeta(x;t)$

$$Eulerian\quad \frac{\partial\varphi}{\partial t}=-g\ \zeta-\frac{1}{2}\left|\nabla\varphi\right|^2-U_0\frac{\partial\varphi}{\partial x}-\frac{U_0^2}{2}-\frac{p_a}{\rho}-damp_1,$$

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$$\frac{d\mathbf{x}}{dt}=\nabla\varphi+U_0\ \delta_{ix}-damp_2\ \delta_{iz}$$

$Semi-Lagrangian$

$$\frac{d\varphi}{dt}=-g\ \zeta-\frac{1}{2}\left|\nabla\varphi\right|^2-U_0\frac{\partial\varphi}{\partial x}-\frac{U_0^2}{2}+\frac{d\zeta}{dt}\cdot\frac{\partial\varphi}{\partial z}-\frac{p_a}{\rho}-damp_1,$$

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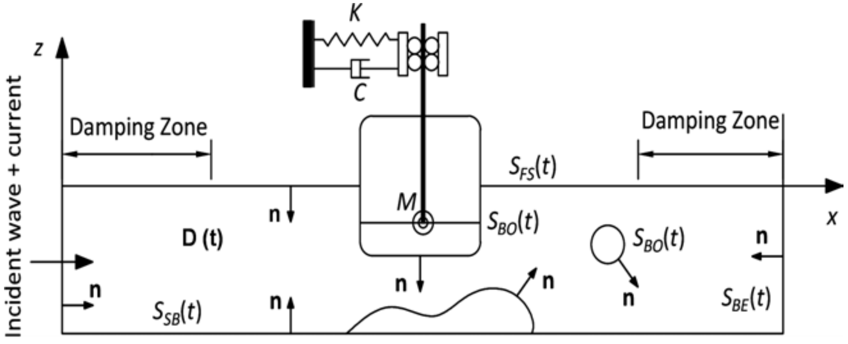


Conditions

- Inlet / Outlet
- Solid boundaries /bottom (zero relative normal velocity

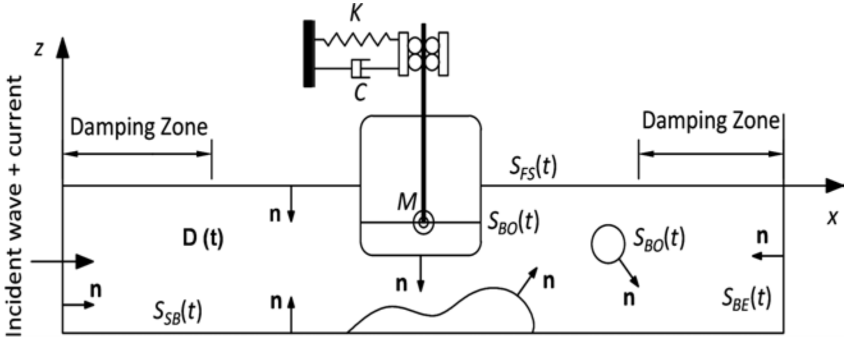
Wave-current-body interaction

Equations

$$\alpha\varphi = \partial_n\varphi * G - \varphi * \partial_n G$$

- Equivalent to translating the Laplacian into a kinematic condition
- The eq "projects "all" of the flow information on FS
- On FS none of the Dirichlet or Neumann data is defined; instead their relation is introduced.
- The eq is solved using piecewise linear distributions & analytic integration
- The implementation is equivalent to projecting the eq using pc basis functions

The same eq is also used for calculating the time derivative of the potential



Details

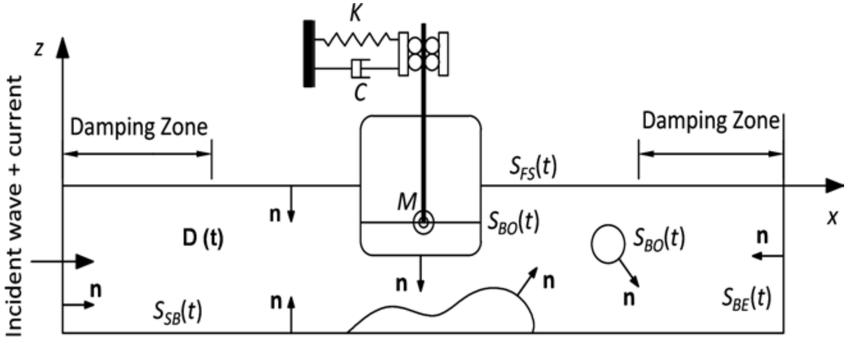
- Double nodes at FS end points

  this results when the inner and outer (incoming) solutions are matched along the inlet in a weak (variational) formulation context.
- Re-griding of the FS (and eventually the solid bodies)
- Inlet: wave introduction through a ramp function

Wave-current-body interaction

Dynamic equations

- Integration in time using RK 4th order
- Semi- and full Lagrangian formulation for the FS were implemented
- + the body motions



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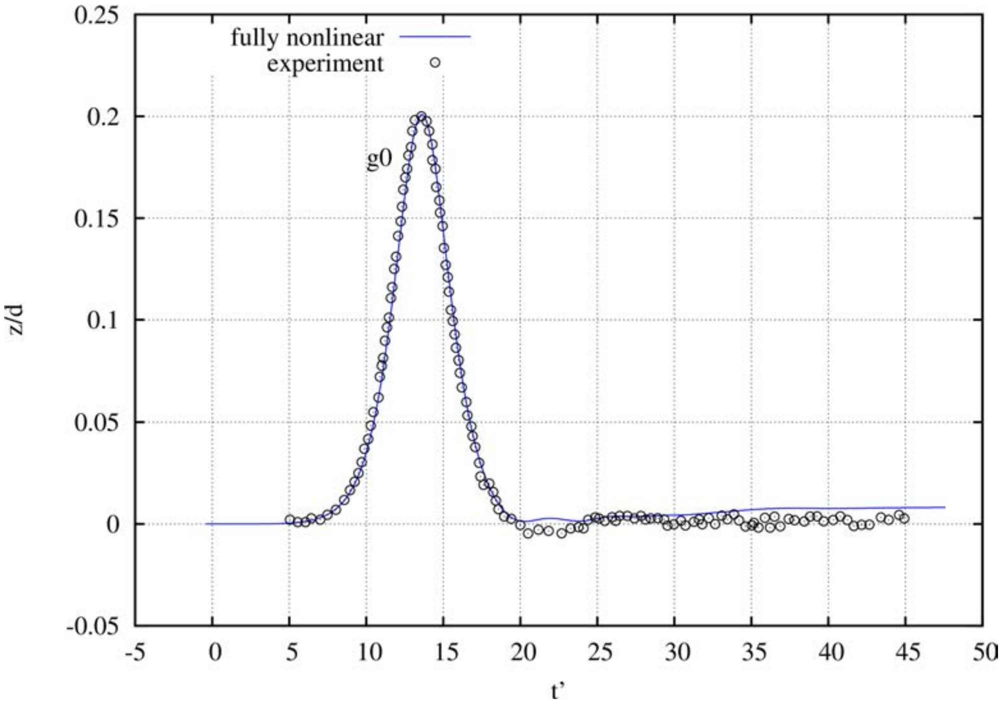
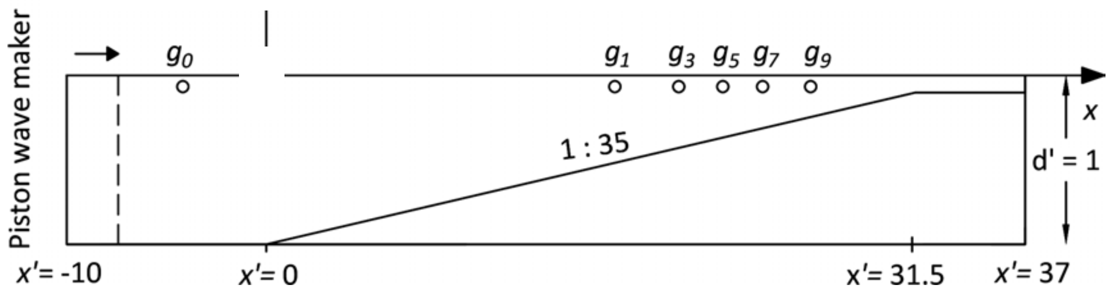
Numerical results

Soliton *H'*=0.2



FS elevation at station g0: x'=-5



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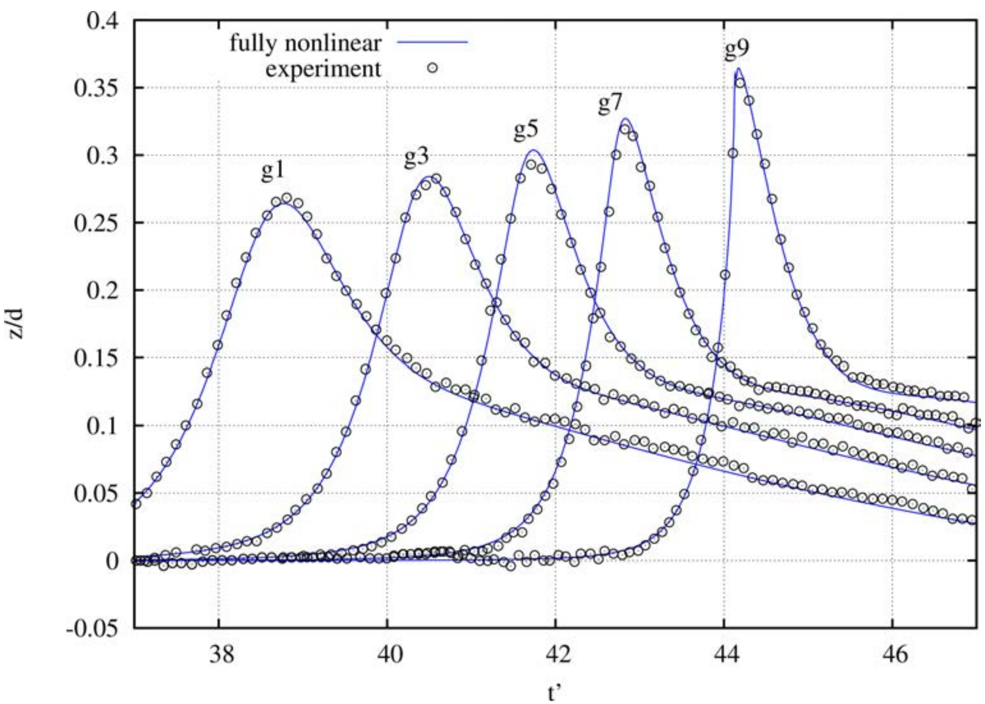
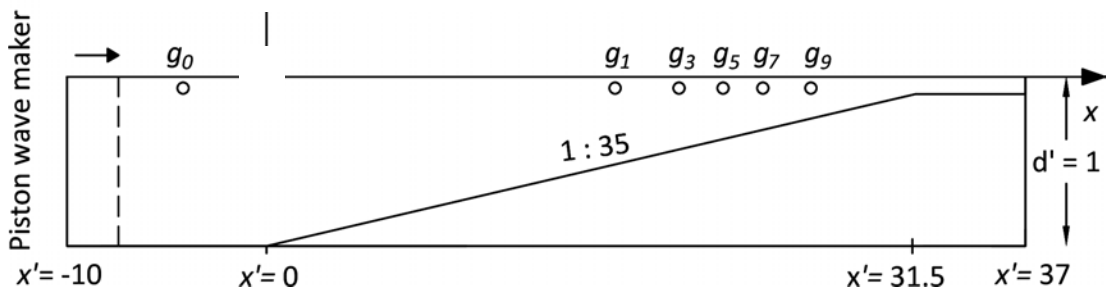
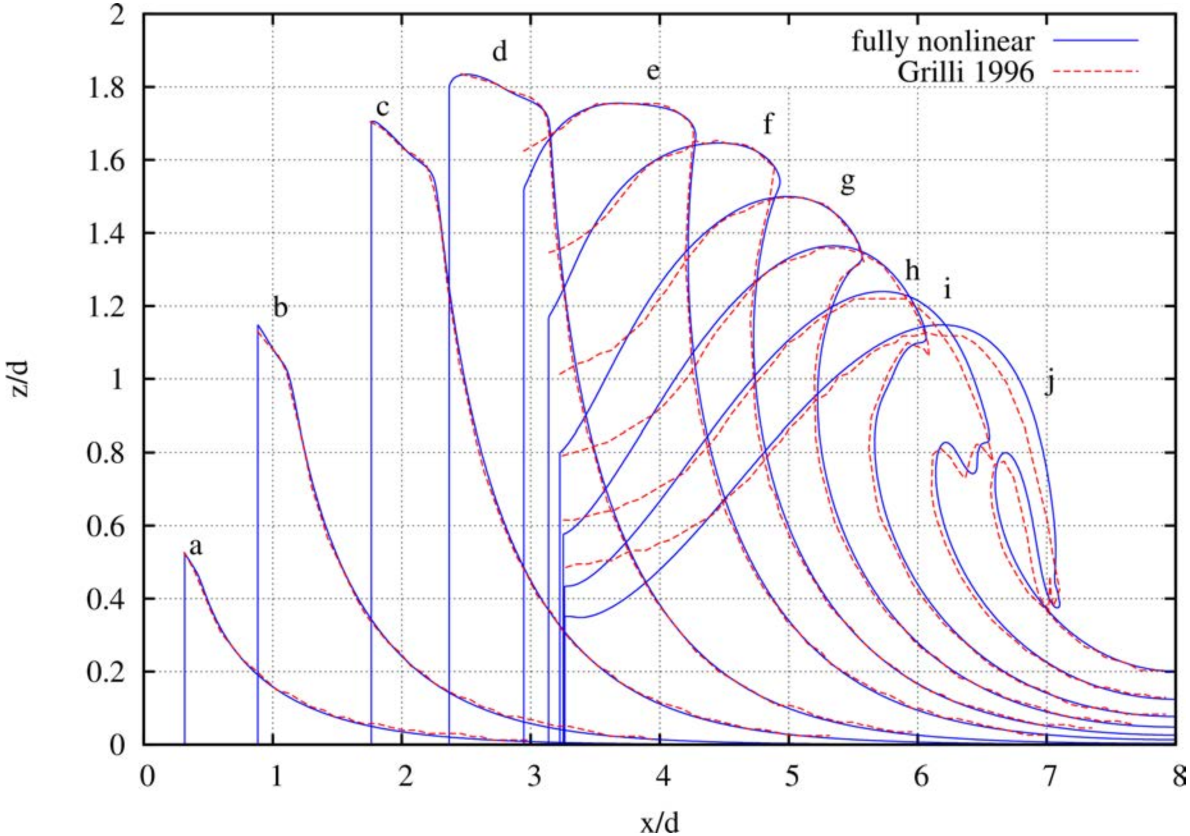


position x'
g0: -5.00
g1: 20.96
g3: 22.55
g5: 23.68
g7: 24.68
g9: 25.91

FS elevation at stations g1-g9

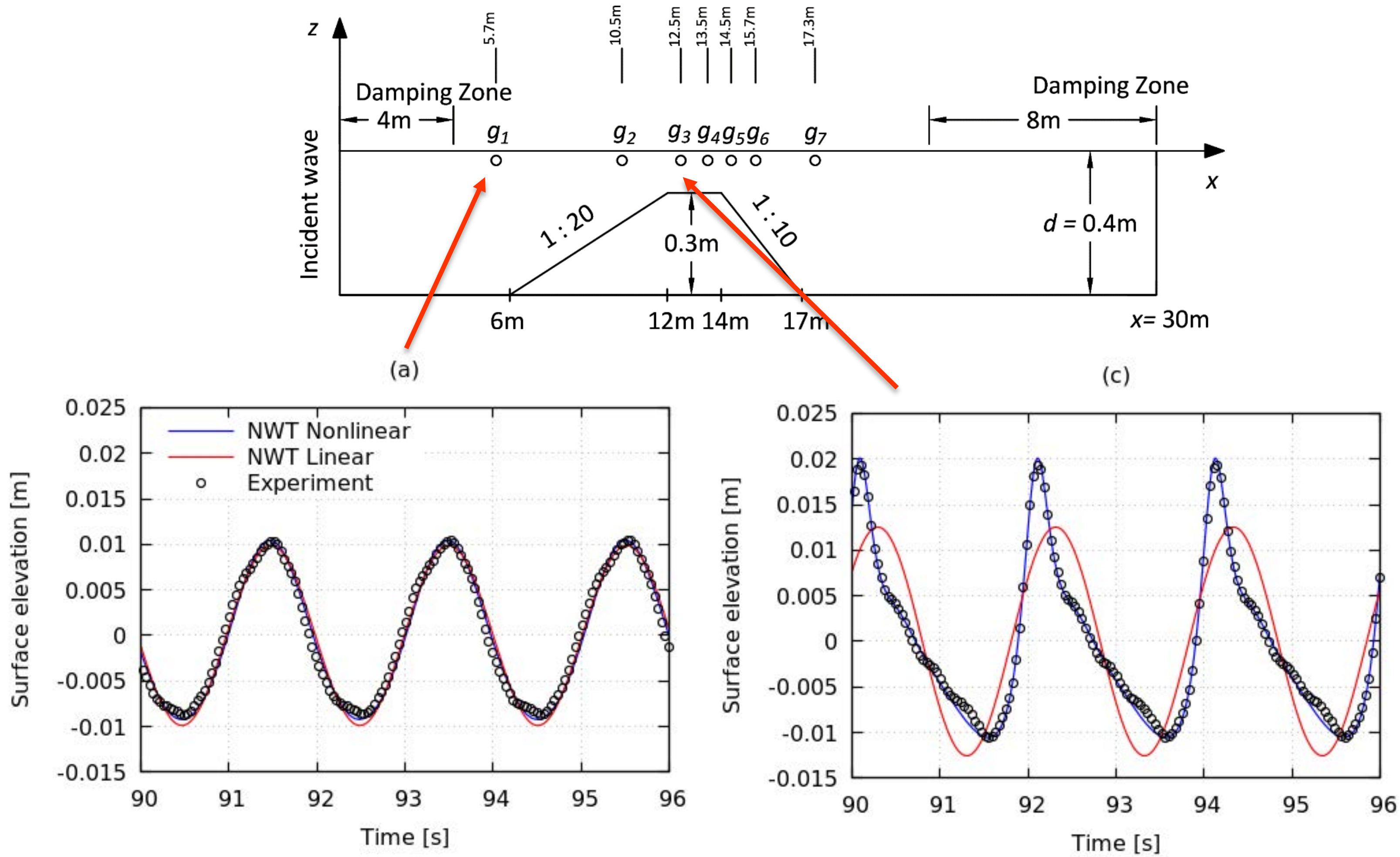


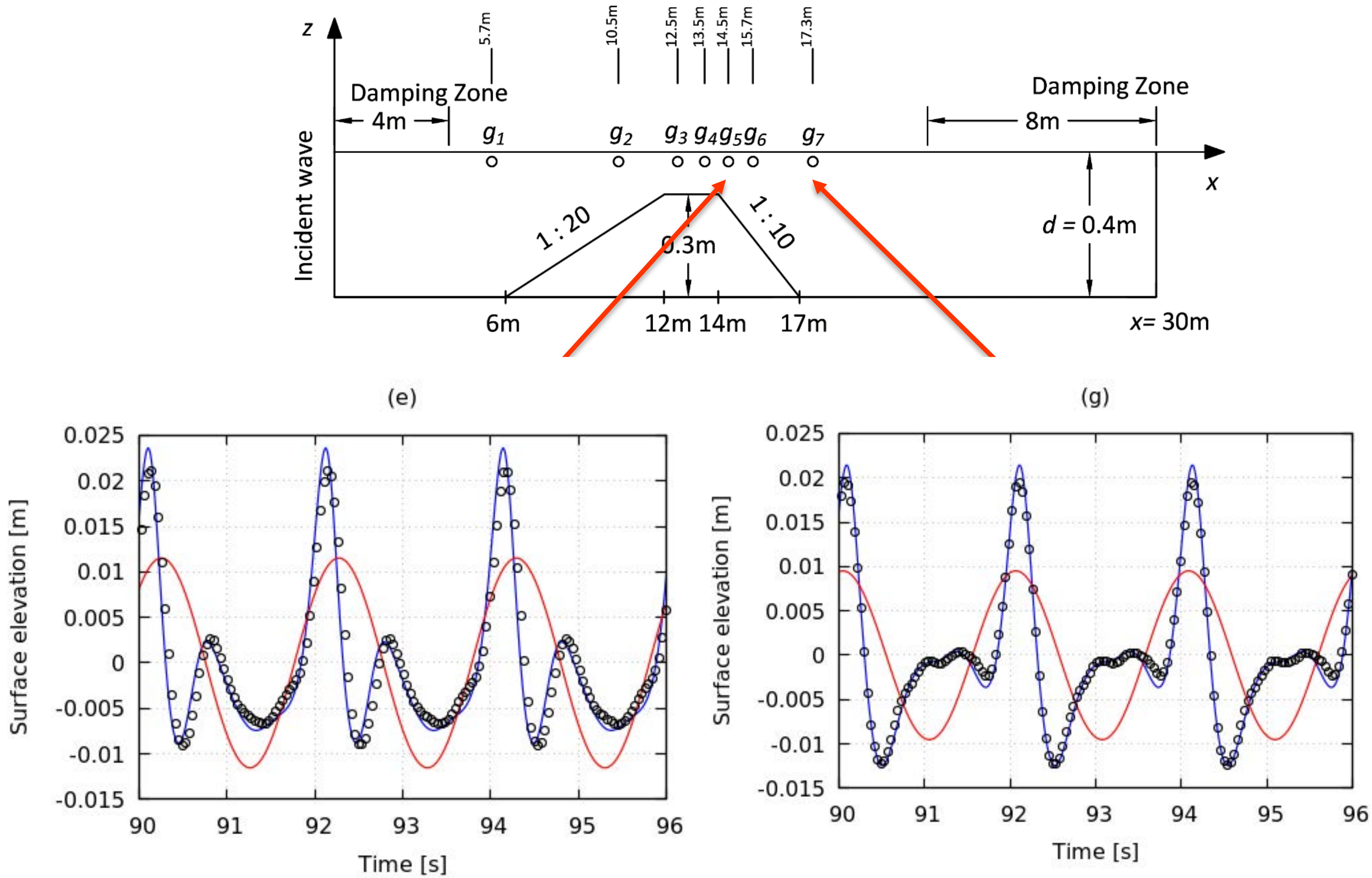
Soliton *H′*=0.2



time t′
a: 2.152
b: 2.776
c: 3.556
d: 4.092
e: 4.724
f: 5.064
g: 5.392
h: 5.648
i: 5.904
j: 6.152

Free-surface snap shots until breaking


z
5.7m
10.5m
12.5m
13.5m
14.5m
15.7m
17.3m
Damping Zone
4m
g1
g2
g3 g4 g5 g6
g7
Damping Zone
8m
x
Incident wave
1 : 20
0.3m
1 : 10
d = 0.4m
6m
12m 14m
17m
x= 30m
(a)
(c)
NWT Nonlinear
NWT Linear
Experiment
Surface elevation [m]
Time [s]
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
90
91
92
93
94
95
96


z
5.7m
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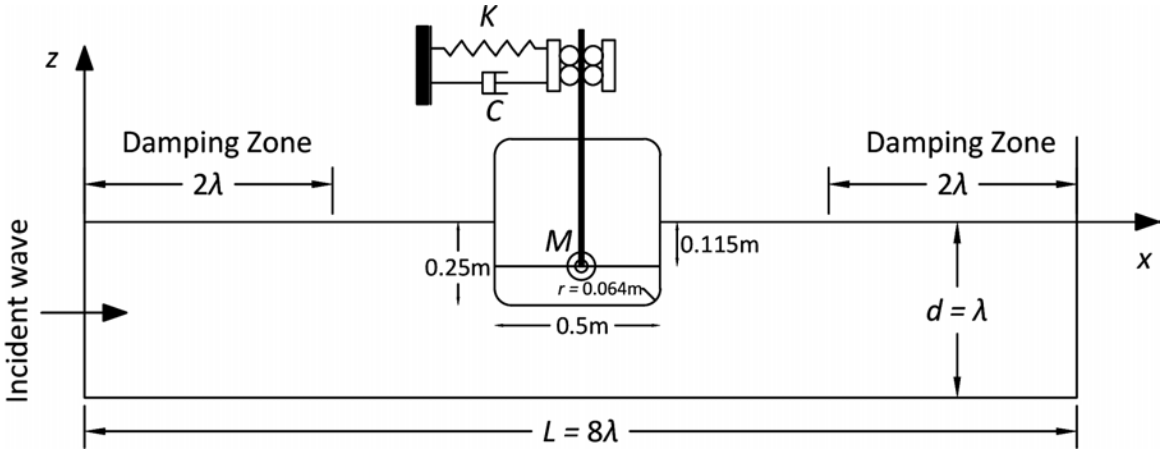
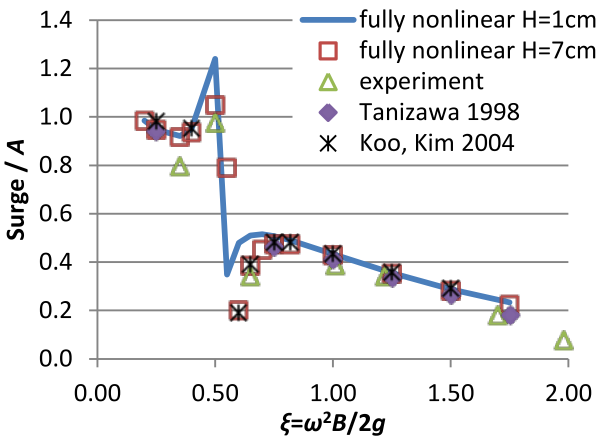
Incident wave
g1
g2
g3 g4 g5 g6
g7
x
1 : 20
0.3m
1 : 10
d = 0.4m
6m
12m 14m
17m
x= 30m
(e)
Surface elevation [m]
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
90
91
92
93
94
95
96
Time [s]
(g)
Surface elevation [m]
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
90
91
92
93
94
95
96
Time [s]

Floating platform

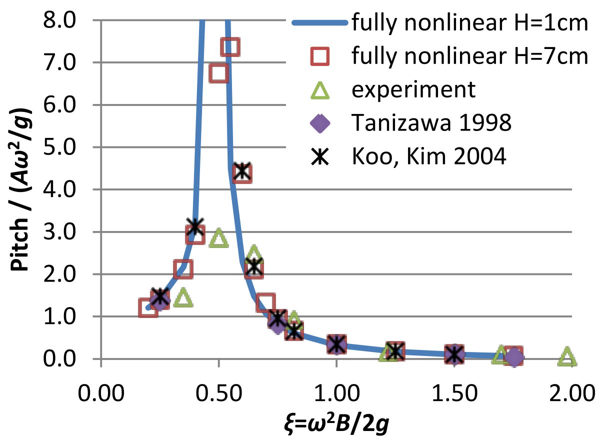


- Conditions: $\xi=\omega^2 B/2=0.2:2$
  H=7cm
- Comparisons with: Experiments [Nojiri - Murayama 1975]
  Predictions [Tanizawa 1998]
  [Koo – Kim 2004, 2007]
  analytic approach [Maruo 1960] (drift force)

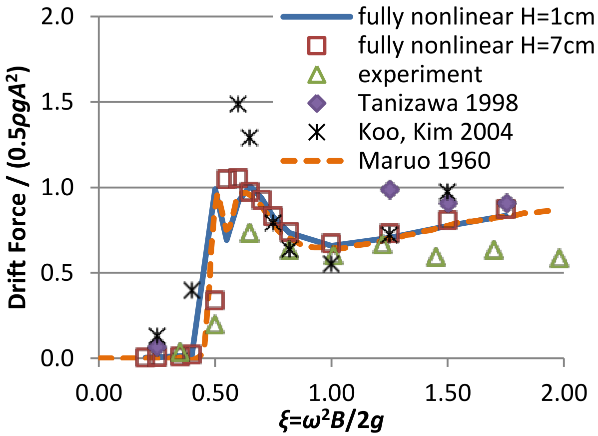
1^{st} harmonic of motions and drift force



Surge - H=7cm



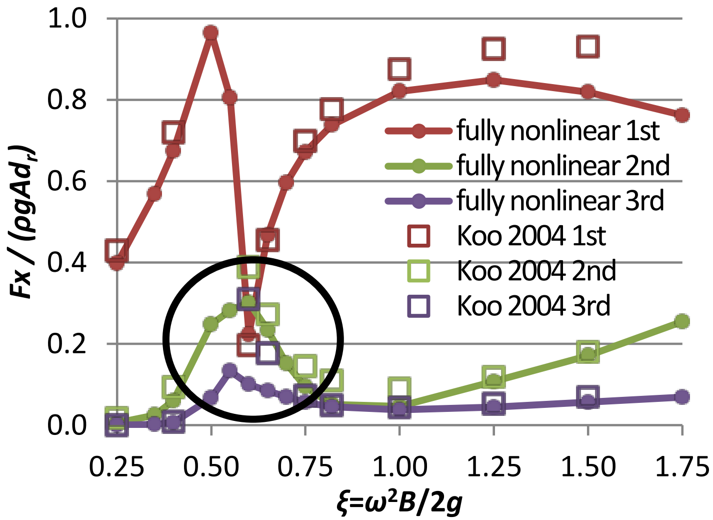
Pitch - H=7cm



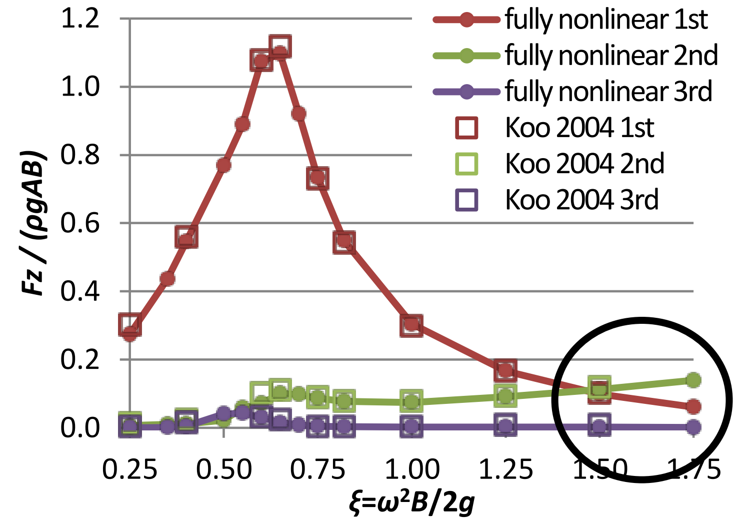
drift force - H=7cm

- ✓ *Consistent results at resonance (up to ϑy~30^o)*
- ✓ *Consistent estimation of drift force (via surface pressure integration)*

Non-linear effects



Horizontal force harmonics



Vertical force harmonics

Onset of non-linear effects:

- *Resonance (horizontal force)*
- *High frequencies (vertical force)*

Outlook

- ✓ Although the specific work did not implement a numerical method by directly using variational formalism, it benefitted a lot in defining its details
- ✓ The specific implementation was successfully tested in a number of cases

The next step is to extend the specific methodology into 3D problems, which will hopefully be done in near future

End of presentation