STOCHASTIC ANALYSIS AND NONLINEAR DYNAMICS

Κύματα, Πιθανότητες, Αναμνήσεις

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RICE

DR. POL D. SPANOS, PROFESSOR
L. B. RYON ENDOWED CHAIR
DEPARTMENT OF CIVIL ENGINEERING

June 30, 2022

Professor Makis Athanasoulis

C/O Professor Themis Sapsis

Referenece: retirement celebration

Dear Makis,

I am extremely sorry that prior rigid constraints are preventing me from participating in the august celebration of your retirement. Nevertheless, with this letter I would like to register my great respect for your contributions as an applied analyst with persistent focusing on uncertainty and nonlinearity issues. I would also like to share my affinity with and appreciation for your witty personality ,and holistic philosophy as a human being. I want to particularly emphasize the long/convoluted scientific discussions that we have had about scientific concepts, and your resolute commitment to substantiveness over impressionism in science and engineering.

I wholeheartedly wish you all the best in whatever you decide to pursue after your retirement. Please note that my group at Rice University will permanently be a welcoming harbor for your scientific trips and trajectories.

Con agape,

Pol

Pol D. Spanos

Pol D Spanos,Ph.D,PE, Fellow:AAM,AvHAA,EMI-ASCE
Medalist/Hon.Member ASME;Dist.Member ASCE
Academician :
NAE(USA);AAAS(USA);AE/EASA/EuAS(Europe);NA(Gr);MEST(TX)
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30 Ιουνίου, 2022

Αγαπητέ Μάκη,

Λυπάμαι πολύ που σοβαροί περιορισμοί με εμποδίζουν να συμμετάσχω στον αξιοσέβαστο εορτασμό για τη συνταξιοδότησή σου. Σε κάθε περίπτωση, με αυτήν την επιστολή θα ήθελα να εκφράσω τον μεγάλο μου σεβασμό για τις συνεισφορές σου ως εφαρμοσμένος αναλυτής με έμφαση σε θέματα αβεβαιότητας και μη γραμμικότητας.

Θα ήθελα επίσης να μοιραστώ τη συμπάθεια και την εκτίμηση για την πνευματώδη προσωπικότητά σου και την ολιστική φιλοσοφία σου ως άνθρωπος. Θέλω να τονίσω ιδιαιτέρως τις μακροχρόνιες/περίπλοκες επιστημονικές συζητήσεις που είχαμε σχετικά με τις επιστημονικές έννοιες και την αποφασιστική δέσμευσή σου για την ουσιαστικότητα έναντι του εντυπωσιασμού στην επιστήμη και το engineering.

Σου εύχομαι ολόψυχα το καλύτερο για ο,τι αποφασίσεις να ακολουθήσεις μετά τη συνταξιοδότησή σου. Σημείωσε ότι η ομάδα μου στο Πανεπιστήμιο Rice θα είναι μόνιμα ένα φιλόξενο λιμάνι για τα επιστημονικά σου ταξίδια και διαδρομές.

Με Αγάπη,

Πολ

STOCHASTIC ANALYSIS AND NONLINEAR DYNAMICS

Extreme event statistics of motions and loads for ships subjected to random waves

Themis Sapsis
Stochastic Analysis and Nonlinear Dynamics Lab
Massachusetts Institute of Technology

Joint work with S. Guth, V. Belenky, K. Weems, V. Pipiras

Some memories

2002… Συναρτησιακή ανάλυση με εφαρμογές στην επιστήμη του μηχανικού
Στοχαστική μοντελοποίηση και πρόβλεψη θαλασσίων συστημάτων

2005…

$$\dot{x}(t)+kx(t)+ax^3(t)=y(t,\omega), \quad x(t_0)=x_0(\omega)$$

Hopf-type equation for the characteristic functional

$$\mathscr{F}(u) = \int_{\mathscr{X}} e^{i\langle u,x\rangle}\mathscr{P}(dx), \qquad u \in \mathscr{X}'$$

$$\frac{d}{dt}\frac{\delta\mathscr{F}_{xy}(u,v)}{\delta u(t)}+k\frac{\delta\mathscr{F}_{xy}(u,v)}{\delta u(t)}-a\frac{\delta^3\mathscr{F}_{xy}(u,v)}{\delta u(t)^3}=\frac{\delta\mathscr{F}_{xy}(u,v)}{\delta v(t)}$$

Joint response-excitation equation

$$\frac{1}{\upsilon}\left.\frac{\partial\phi_{xy}(\upsilon,t;\nu;s)}{\partial t}\right|_{s=t} + k\frac{\partial\phi_{xy}(\upsilon,t;\nu,t)}{\partial\upsilon} - a\frac{\partial^3\phi_{xy}(\upsilon,t;\nu,t)}{\partial\upsilon^3} = \frac{\partial\phi_{xy}(\upsilon,t;\nu,t)}{\partial\nu}$$

$$\phi_{xy}(0,t;\nu,t)=\phi_y(\nu,t), \quad \nu\in\mathbb{R}, \quad t\geq t_0$$

$$\phi_{xy}(\upsilon,t_0;0,t_0)=\phi_x(\upsilon,t_0)=\phi_0(\upsilon), \quad \upsilon\in\mathbb{R}$$

Waves, (Ships), and Probability

The goal

hard

A link between: **Geometry ↔ Dynamics ↔ Extreme Event Statistics**

The challenge

This is a hard, but nevertheless, solved problem if the involved processes are Gaussian or the dynamics are linear.

This is an open problem if the dynamical system is nonlinear. For ship motions/loads the underlying system is often non-linear and the response statistics non-Gaussian.

Extreme value properties of loads…

→ Spectrum-based irregular waves in naval architecture by St. Denis and Pierson (1953)

→ Weibull distribution typical for computing lifetime extreme wave-induced loads since 1960

→ Extreme value theory employed for calculation of wave-induced loads, Ochi and Wang (1976)

The plan

A) Exactly solvable reduced order models

Go beyond Gaussian statistics or Extreme Value PDFs…

i) Design models which are complex enough to capture the observed non-Gaussian features and simple enough to be analytically solvable.

ii) Interpret observed statistical complexity (from direct simulations) in terms of the hull geometrical properties.

iii) Understand the limitations of these analytically solvable models.

B) Computational method using carefully designed wave-episodes

Extreme event statistics require huge number of samples

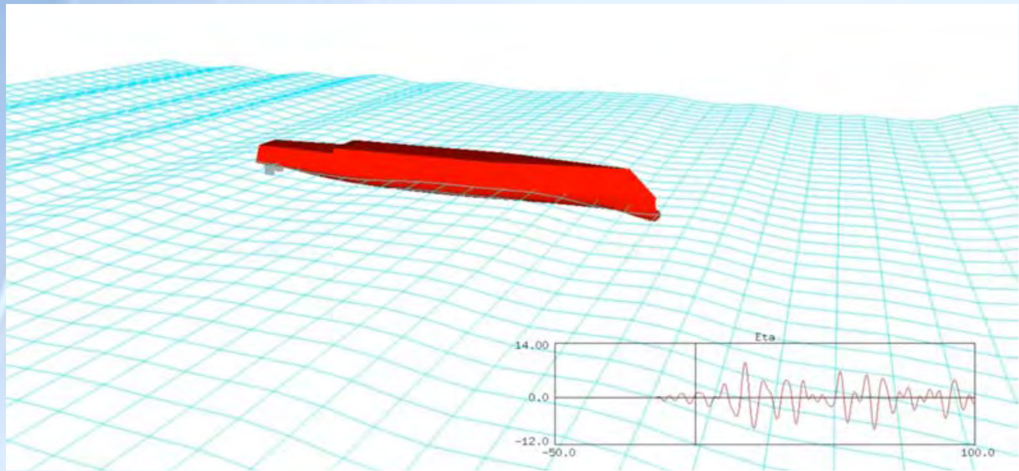
i) Random waves are associated with high dimensional spaces

ii) Formulate a rapidly converging representation using wave-episodes

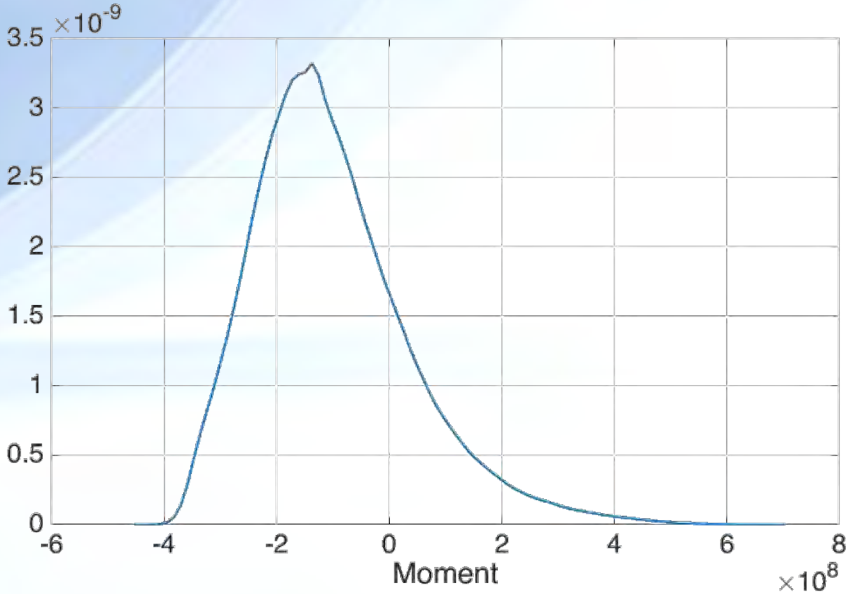
iii) Express the ship response/loads through GPR.

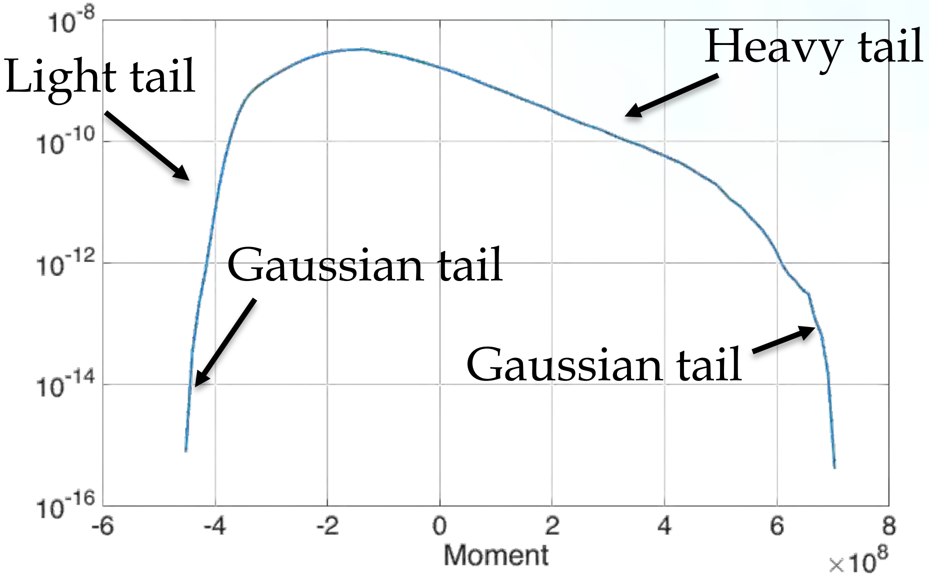
iv) Careful modeling of transient features

Extreme event statistics for structural loads: Observations

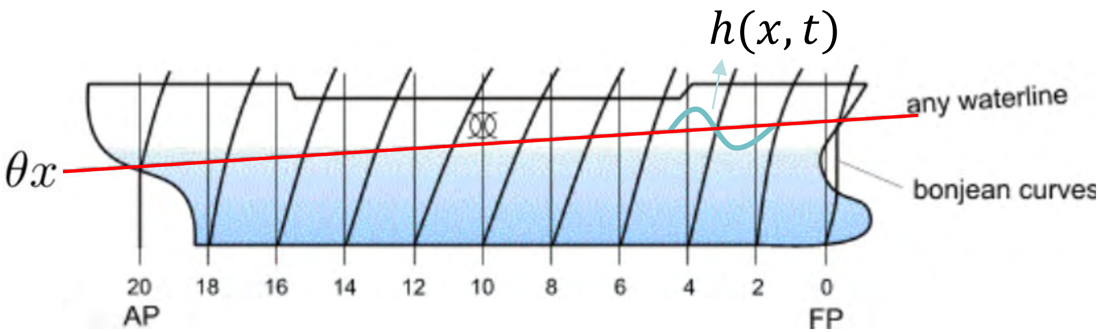


Pdf for bending moment at mid-ship from direct numerical simulations





Development of analytical model for VBM



$$A(x, \theta x + h(x,t))$$

Area below station x for pitch angle θ and local wave elevation $h(x,t)$

Froude-Krylov Hydrostatic moment

$$M_{FKHS}(\theta; h(x,t;\zeta)) = \int_{-L/2}^{L/2} xA(x, \theta x + h(x,t;\zeta))dx$$

Expansion of area curves around pitch-WL (or wave-averaged WL)

$$A(x, \theta x + h) = A(x, \theta x) + \sum_{q=1}^{Q} \frac{A^{(q)}(x, \theta x)}{q!} h^q$$

$$M_{FKHS}(\theta, t; \zeta) = \int_{-L/2}^{L/2} xA(x, \theta x)dx + \sum_{q=1}^{Q} \int_{-L/2}^{L/2} xA^{(q)}(x, \theta x) \frac{h(x,t;\zeta)^q}{q!} dx$$

Gaussian white-noise approximation of the wave excitation

$$M_{FKHS}(\theta,t;\zeta)=\int_{-L/2}^{L/2} xA(x,\theta x)dx+\sum_{q=1}^{Q}\int_{-L/2}^{L/2} xA^{(q)}(x,\theta x)\frac{h(x,t;\zeta)^q}{q!}dx$$

Mean of the FKH moment (Q=1)

$$\mathbb{E}^{\zeta}[M_{FKHS}(\theta,t;\zeta)]=\underbrace{\int_{-L/2}^{L/2} xA(x,\theta x)dx}_{M_{HS}(\theta)}$$

$M_{HS}(\theta)$ → Hydrostatic component

Zero – mean stochastic part of the FKH moment

$$\int_{-L/2}^{L/2} xA'(x,\theta x)h(x,t;\zeta)dx\cong -D\dot{W}$$

- White-noise approximation of the stochastic moment
- Assumes rapidly decorrelating wave field in time

Approximate equation of pitch motion

$$I\ddot{\theta}+c\dot{\theta}+M_{HS}(\theta)=D\dot{W}$$

- Assumes first order effects for mean and stochastic component

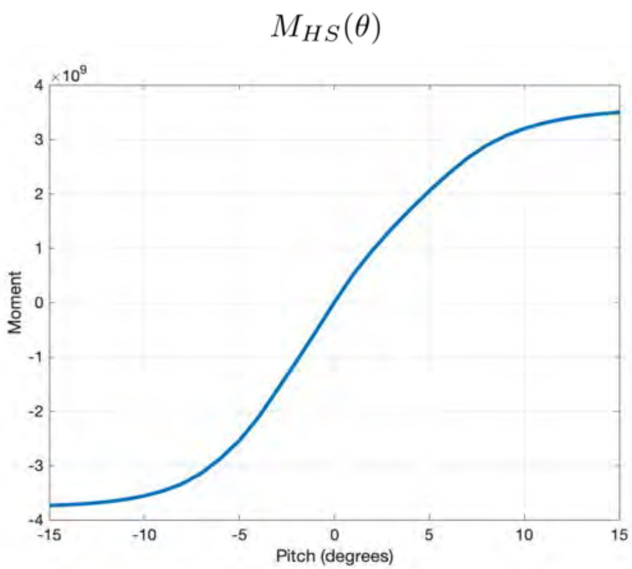
Development of analytical model for VBM

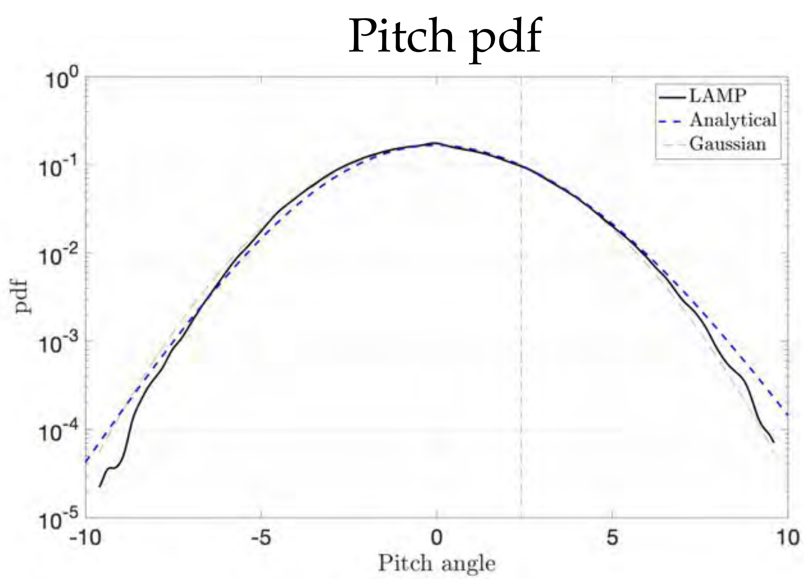
Analytical solution of pitch pdf

$$I\ddot{\theta} + c\dot{\theta} + M_{HS}(\theta) = D\dot{W}$$

Nonlinearity enters through hydrostatics

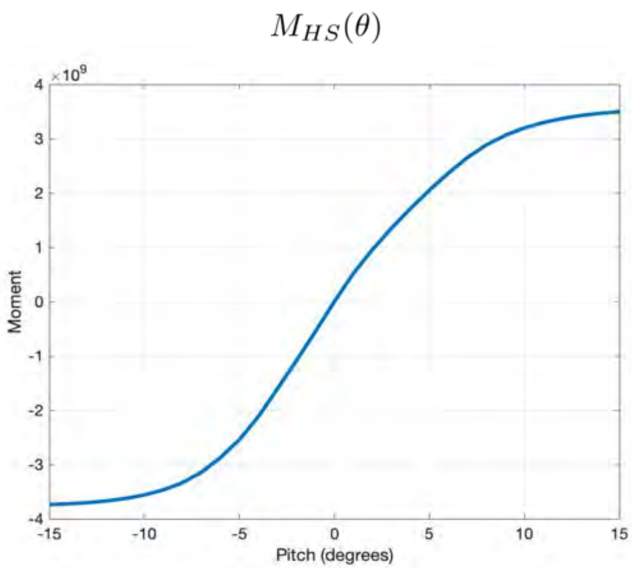
$$\boxed{\begin{aligned} p_s(\theta, \dot{\theta}) &= C \exp\left(-\frac{c}{2D^2}\left(I\dot{\theta}^2 + V_{HS}(\theta)\right)\right) \\ V_{HS}(\theta) &= \int M_{HS}(\theta) d\theta \end{aligned}}$$

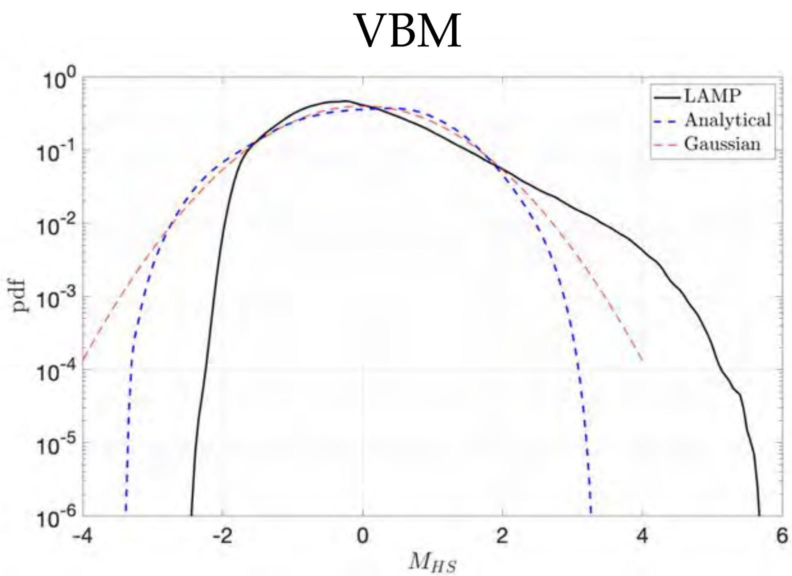




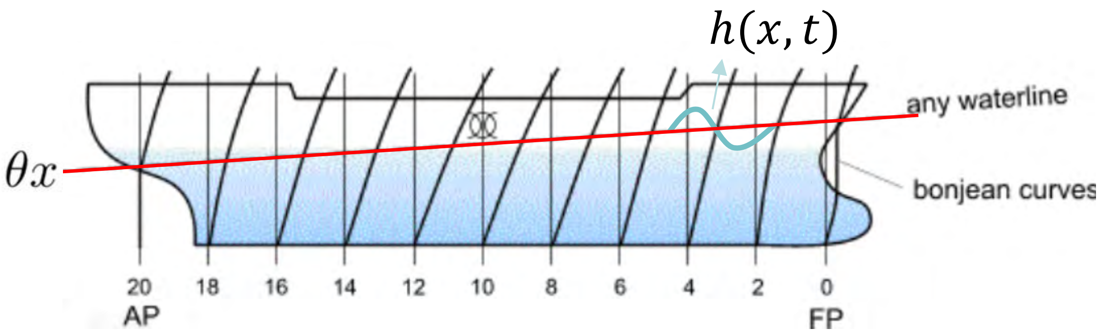
Development of analytical model for VBM

$$M_{FKHS}(\theta, t; \zeta) = \int_0^{L/2} xA(x, \theta x)dx$$





Development of analytical model for VBM



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Development of analytical model for VBM

Utilize a monochromatic approximation for the waves:

$$h(x,t) = a_c(t)\cos\left(\frac{2\pi x}{L}\right) + a_s(t)\sin\left(\frac{2\pi x}{L}\right)$$

Similar to Grim effective wave approach

$$M_{FKHS}(\theta,t;\zeta) = \int_{-L/2}^{L/2} xA(x,\theta x)dx + \sum_{q=1}^{Q}\int_{-L/2}^{L/2} xA^{(q)}(x,\theta x)\frac{h(x,t;\zeta)^q}{q!}dx$$

$Q = 2$

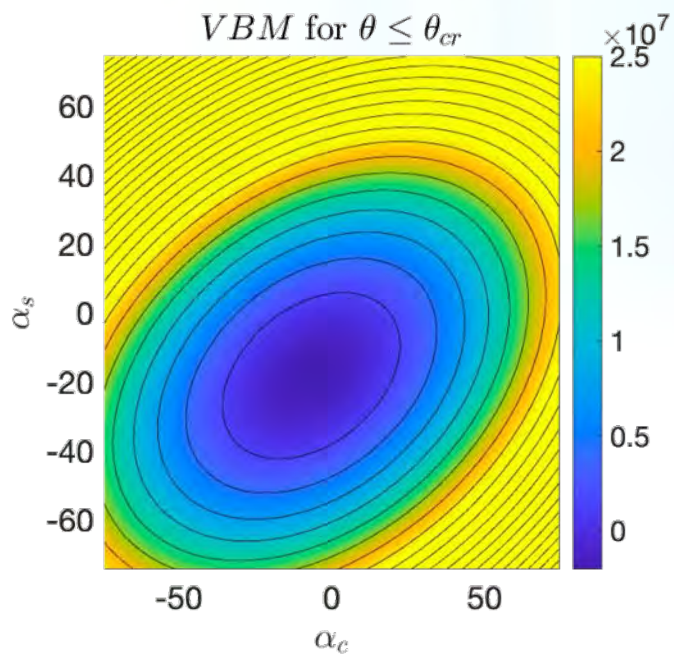
Approximation of the VBM in terms of the wave amplitudes (from FK)

$$M_{W-VBM}(\alpha_c,\alpha_s) = \overline{\rho_c(\theta)}a_c(t) + \overline{\rho_s(\theta)}a_s(t) + \overline{\rho_{c^2}(\theta)}\alpha_c^2(t) + \overline{\rho_{s^2}(\theta)}\alpha_s^2(t) + \overline{\rho_{cs}(\theta)}\alpha_s(t)\alpha_c(t).$$

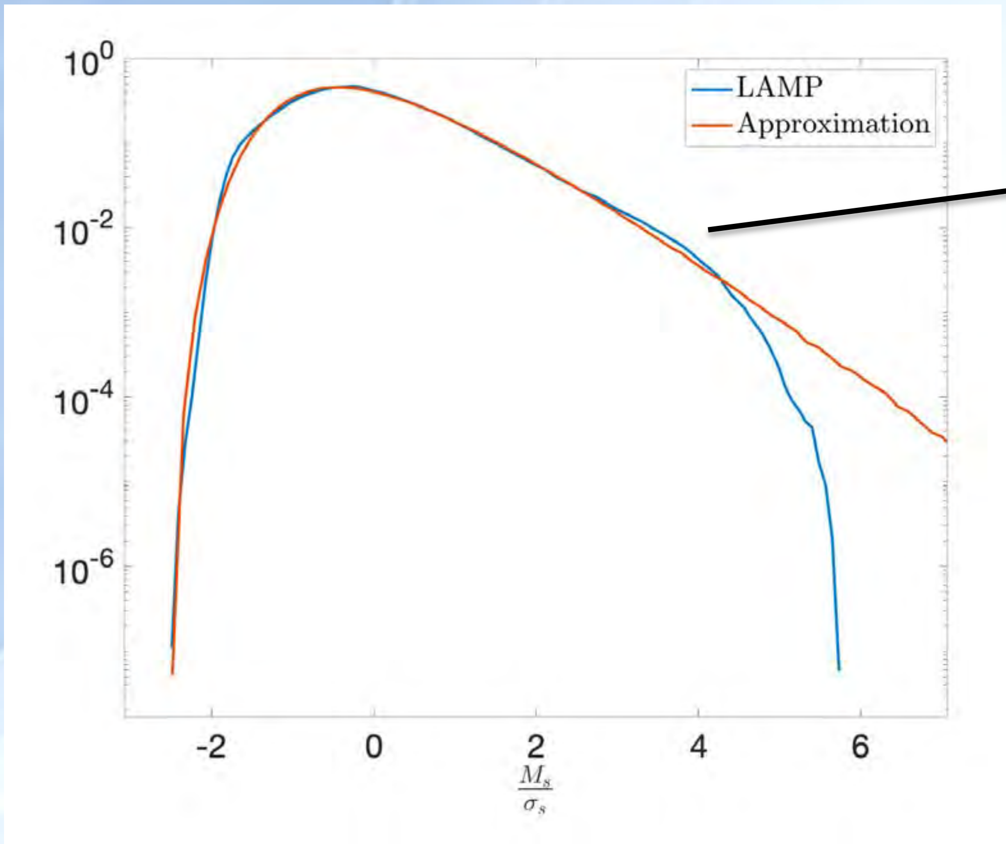
$$\rho_c(\theta) = \int_0^{L/2} xA'(x,\theta x)\cos\left(\frac{2\pi x}{L}\right)dx, \quad \text{and} \quad \rho_s(\theta) = \int_0^{L/2} xA'(x,\theta x)\sin\left(\frac{2\pi x}{L}\right)dx$$

$$\rho_{c^2}(\theta) = \frac{1}{2}\int_0^{L/2} xA''(x,\theta x)\cos^2\left(\frac{2\pi x}{L}\right)dx, \quad \rho_{s^2}(\theta) = \frac{1}{2}\int_0^{L/2} xA''(x,\theta x)\sin^2\left(\frac{2\pi x}{L}\right)dx,$$

$$\rho_{cs}(\theta) = \int_0^{L/2} xA''(x,\theta x)\cos\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx.$$



Analytical approximation of the VBM (no deck) due to FK



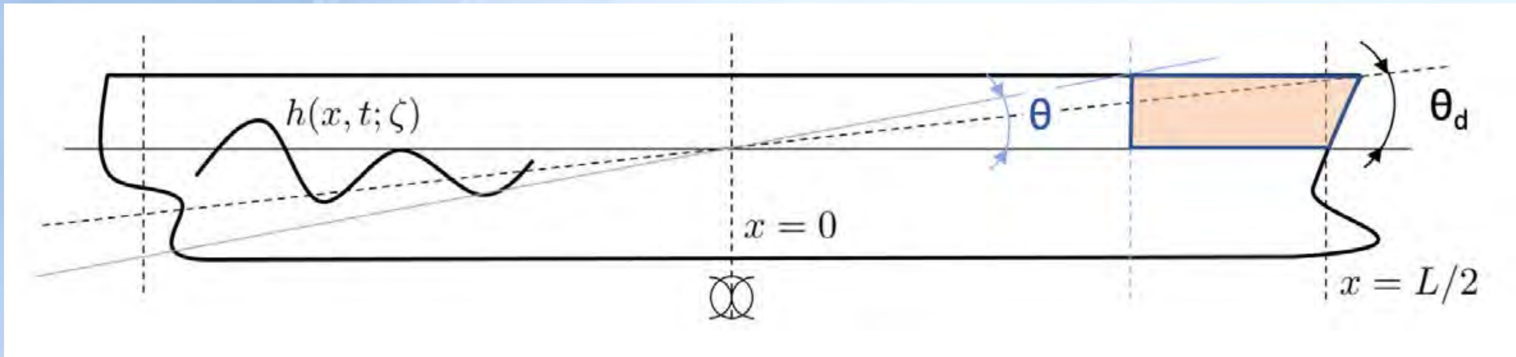
Divergence for large positive moments

Tunable parameter: variance of the wave excitation

Sapsis et al., 33rd SNH, 2020

Critical pitch angle due to deck submergence

Computation of the effective VBM for the case of deck submergence



$$\rho_{c^k s^l}(\theta) = \frac{1}{k!l!}\int_0^{\frac{L\tan\theta_d}{2\tan\theta}} x \left.\frac{\partial^{k+l}A(x,z)}{\partial z^{k+l}}\right|_{z=x\theta} \cos^k\left(\frac{2\pi x}{L}\right)\sin^l\left(\frac{2\pi x}{L}\right)dx, \quad \theta \geq \theta_d$$

Wave induced VBM, when deck is submerged:

$$M_{W-VBM}(\theta) = \frac{\tan\theta_d}{\tan\theta} M_{W-VBM}(\alpha_c,\alpha_s), \quad \theta \geq \theta_d$$

Critical angle is defined as the one for which

$$M_{W-VBM}(\theta_{cr}) = qM_{W-VBM}(\alpha_c,\alpha_s)$$

$0 < q < 1$ is a parameter

$$\theta_{cr} = \theta_d / q$$

$$\theta_d = 5.87\,\text{deg}$$

$$\theta_{cr} = 13.5\,\text{deg}$$

How do we account for the finite deck?

1) Derive critical pitch beyond witch no additional VBM is contributing

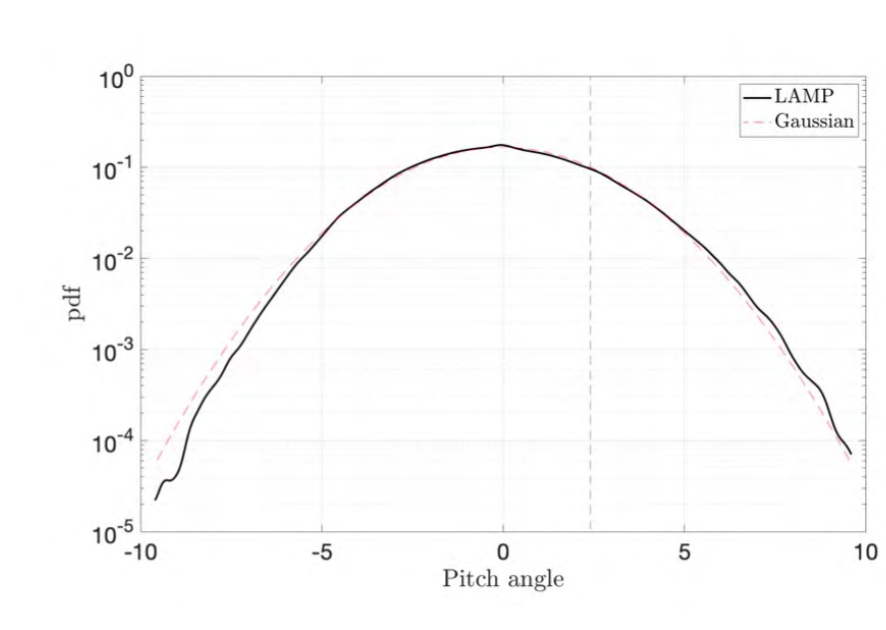
2) Derive correlation between waves and pitch statistics.

3) Derive modified pdf for VBM with deck effects

Correlation between waves and pitch

Need to condition on waves that result in subcritical pitch angles.

How to identify these waves?



Pitch statistics follow a normal distribution →

Develop a linear (Gaussian) model

Linear model allows for analytical characterization of correlation structure

Correlation between waves and pitch

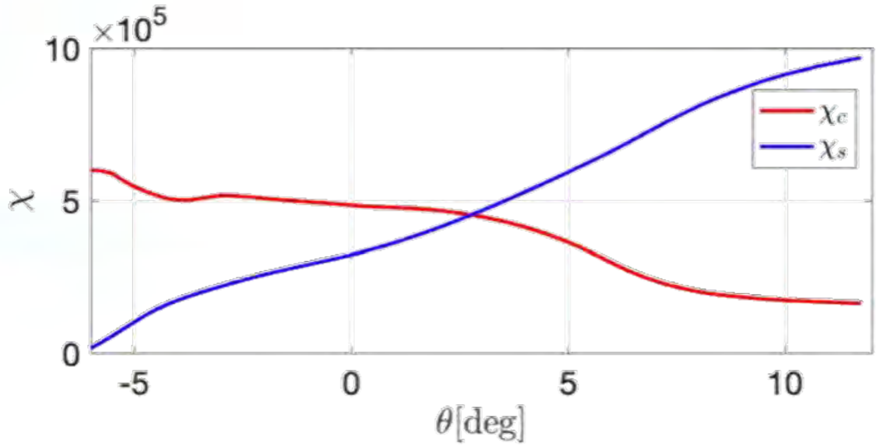
Expanding Froude Krylov forces up to linear terms (Gaussian approximation):

$$M_{FK}(\theta, t; \zeta) = \int_{-L/2}^{L/2} xA'(x, \theta x) h(x, t; \zeta) dx$$

We have the equation of motion for pitch…

$$I\ddot{\theta} + c\dot{\theta} + k\theta = a_c(t)\chi_c(\theta) + a_s(t)\chi_s(\theta)$$

$$\chi_c(\theta) = \int_{-L/2}^{L/2} xA'(x, \theta x) \cos\left(\frac{2\pi x}{L}\right) dx, \quad \text{and} \quad \chi_s(\theta) = \int_{-L/2}^{L/2} xA'(x, \theta x) \sin\left(\frac{2\pi x}{L}\right) dx$$



ONR
Topsides
Fared hull

Correlation between waves and pitch

Averaging the hydrodynamic coefficients over θ

Stochastic amplitudes

$$I\ddot{\theta} + c\dot{\theta} + k\theta = a_c(t)\bar{\chi}_c + a_s(t)\bar{\chi}_s$$

$$\overline{\chi(\theta)} = \int_{-\infty}^{\infty} \chi(\theta) \frac{1}{\sqrt{2\pi\sigma_\theta^2}} \exp\left(-\frac{\theta^2}{2\sigma_\theta^2}\right) d\theta$$

(in our numerical example $\sigma_\theta = 2.4009$)

Employ derived stochastic model with Wiener-Khinchin relations:

$$S_{\theta a_c}(\omega) = H_{\theta a_c}(\omega) S_{a_c a_c}(\omega) \quad \text{and} \quad S_{\theta a_s}(\omega) = H_{\theta a_s}(\omega) S_{a_s a_s}(\omega),$$

with

Wave spectrum

$$H_{\theta a_c}(\omega) = \frac{\bar{\chi}_c}{k - I\omega^2 + ic\omega} \quad \text{and} \quad H_{\theta a_s}(\omega) = \frac{\bar{\chi}_s}{k - I\omega^2 + ic\omega},$$

Correlation between waves and pitch

In this way we obtain…

$$\boxed{\begin{aligned} C_{\theta a_c} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{\chi}_c S_0(\omega)d\omega}{k - I\omega^2 + ic\omega} = \bar{\chi}_c\mathcal{S} \\ C_{\theta a_s} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{\chi}_s S_0(\omega)d\omega}{k - I\omega^2 + ic\omega} = \bar{\chi}_s\mathcal{S} \end{aligned}}$$

with $\mathcal{S} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{S_0(\omega)d\omega}{k - I\omega^2 + ic\omega}$

From moment equations we obtain

$$\mathcal{S} = \frac{k\sigma_\theta^2 - I\sigma_{\dot{\theta}}^2}{\bar{\chi}_c^2 + \bar{\chi}_s^2}$$

Conditional wave statistics for deck submergence

The pdf for the wave amplitudes $\vec{\alpha} = (\alpha_c, \alpha_s)$ is assumed to be Gaussian

$$f(\alpha_c, \alpha_s) = \frac{1}{2\pi\sigma_a^2} \exp\left(-\frac{\alpha_c^2 + \alpha_s^2}{2\sigma_\alpha^2}\right)$$

<u>Conditional pdf for the waves that cause pitch angles smaller than the critical one</u>

Using Bayes rule we obtain

$$f(\alpha_c, \alpha_s | \theta \leq \theta_{cr}) = \frac{P(\theta \leq \theta_{cr} | \alpha_c, \alpha_s) f(\alpha_c, \alpha_s)}{P(\theta \leq \theta_{cr})}$$

where,

$$P(\theta \leq \theta_{cr} | \alpha_c, \alpha_s) = \int_{-\infty}^{\theta_{cr}} f(\theta | \alpha_c, \alpha_s) d\theta$$

Conditional pdf for pitch with given waves

Conditional wave statistics for deck submergence

$$P(\theta \leq \theta_{cr}|\alpha_c, \alpha_s) = \int_{-\infty}^{\theta_{cr}} f(\theta|\alpha_c, \alpha_s)d\theta$$

Gaussian

Conditional mean $$\bar{\theta}_{\alpha_c,\alpha_s} = \frac{C_{\theta a_c}}{\sigma_\alpha^2}\alpha_c + \frac{C_{\theta a_s}}{\sigma_\alpha^2}\alpha_s$$

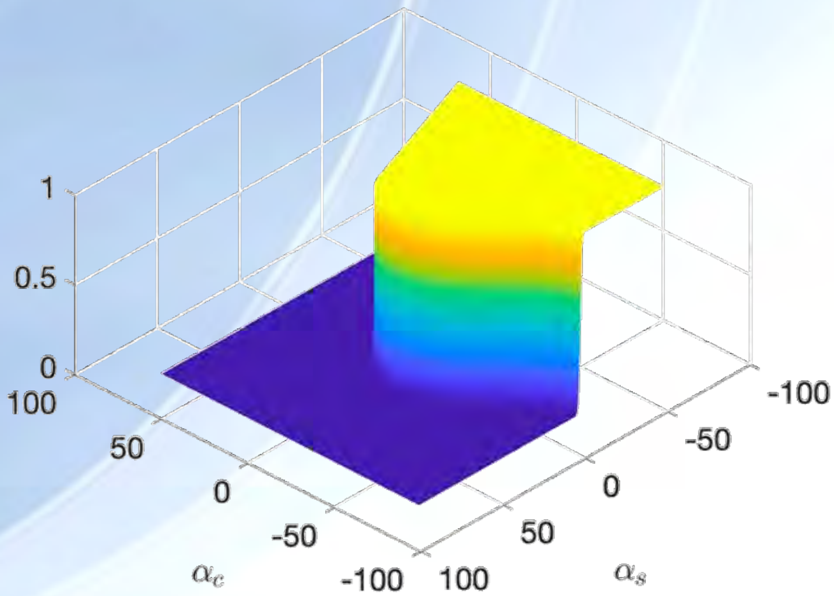
Conditional covariance $$\sigma^2_{\theta|\alpha_c,\alpha_s} = \sigma_\theta^2 - \frac{C^2_{\theta a_c}}{\sigma_\alpha^2} - \frac{C^2_{\theta a_s}}{\sigma_\alpha^2}$$

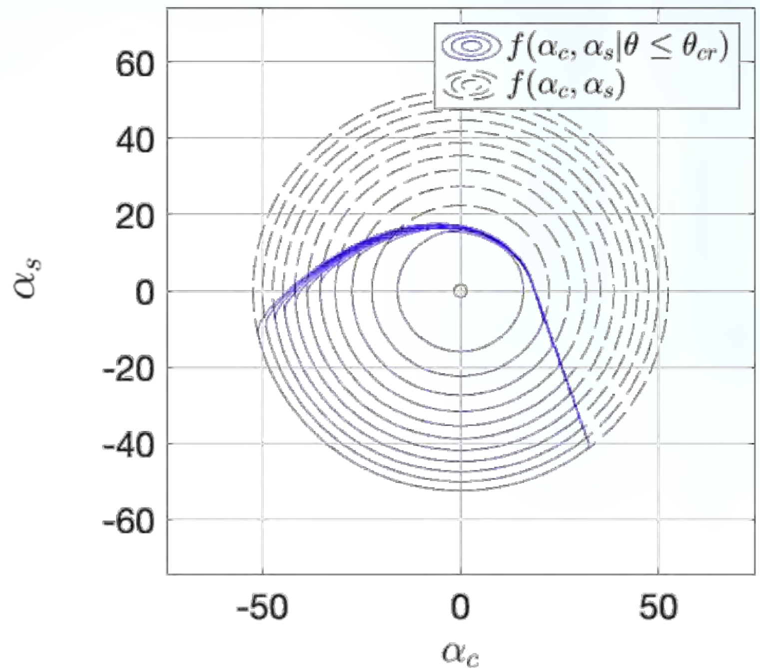
$$P(\theta \leq \theta_{cr}|\alpha_c, \alpha_s) = \int_{-\infty}^{\theta_{cr}} f(\theta|\alpha_c, \alpha_s)d\theta = \Phi\left[\frac{\theta_{cr} - \bar{\theta}_{\alpha_c,\alpha_s}}{\sigma_{\theta|\alpha_c,\alpha_s}}\right]$$

Conditional wave statistics for deck submergence

$$P(\theta \leq \theta_{cr}|\alpha_c, \alpha_s) = \Phi\left[\frac{\theta_{cr} - \bar{\theta}_{\alpha_c,\alpha_s}}{\sigma_{\theta|\alpha_c,\alpha_s}}\right]$$

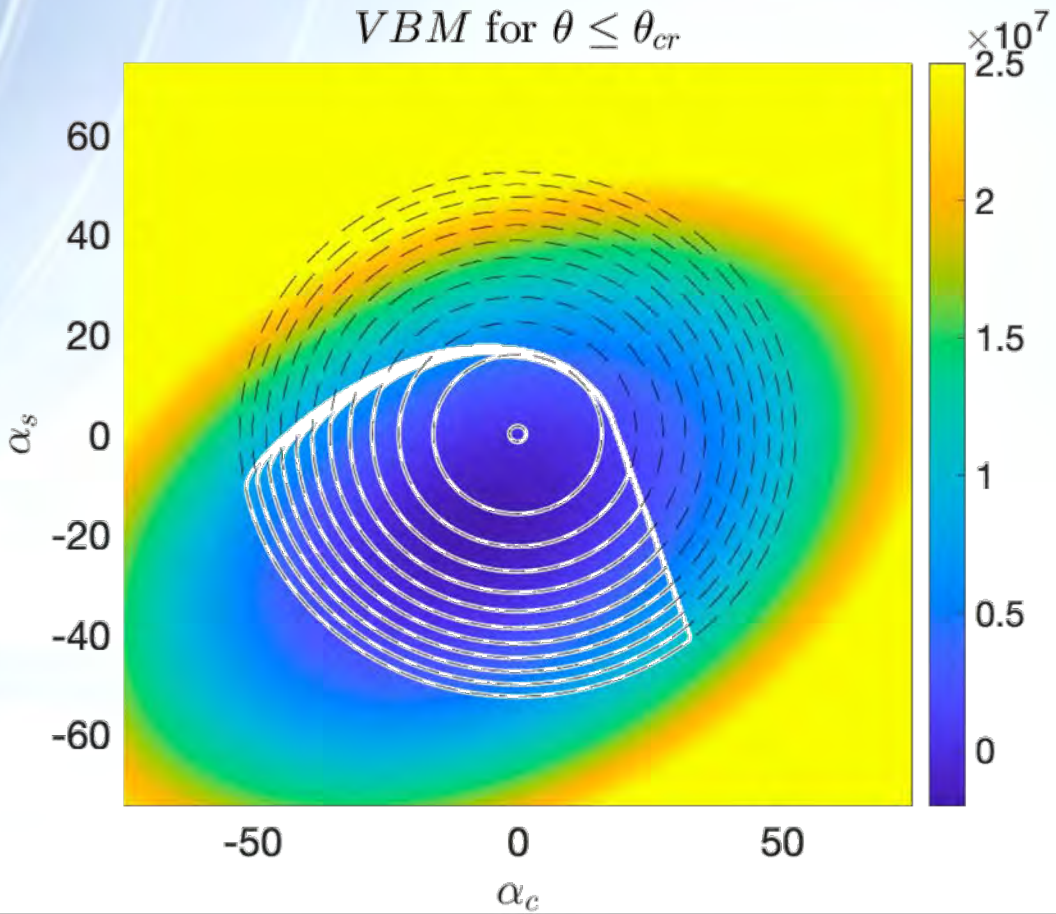
$$f(\alpha_c, \alpha_s|\theta \leq \theta_{cr}) = \Phi\left[\frac{\theta_{cr} - \bar{\theta}_{\alpha_c,\alpha_s}}{\sigma_{\theta|\alpha_c,\alpha_s}}\right]\frac{f(\alpha_c, \alpha_s)}{P(\theta \leq \theta_{cr})}$$





Pdf for wave induced VBM with deck effects

$$F^{deck}_{M_{W-VBM}}(M) = \int\limits_{D(M)} f(\alpha_c, \alpha_s | \theta \leq \theta_{cr}) d\alpha_c d\alpha_s, \quad D(M) = \{(\alpha_c, \alpha_s) : M_{W-VBM}(\alpha_c, \alpha_s) \leq M\}$$



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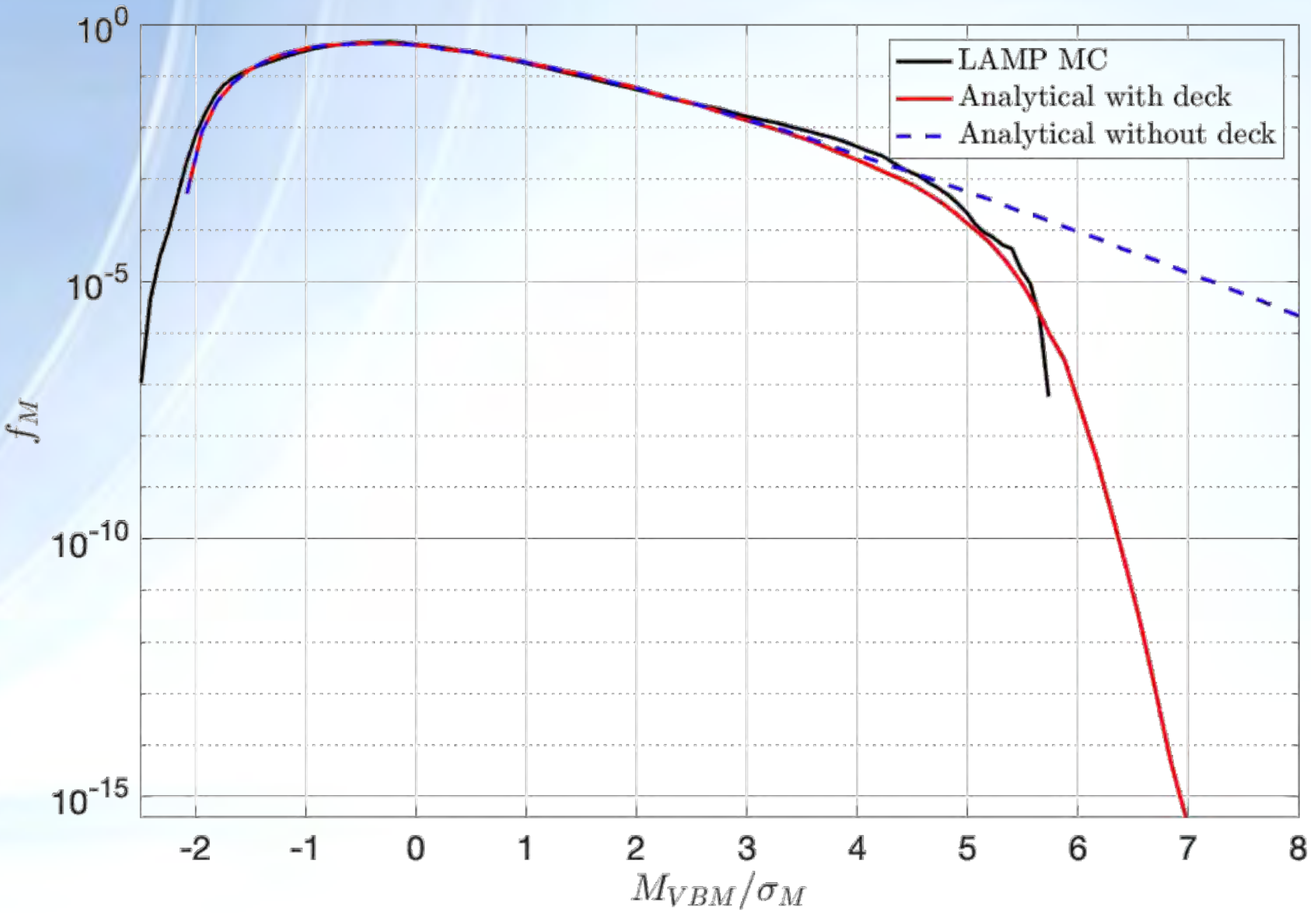


Sapsis et al., 34th SNH, 2022

Traditional computational methods for extreme event statistics

Frequency Domain: **Weiner-Khintchine Relation**:

$$S_{MM}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$$

- Use the **transfer function** to relate statistics of sea surface to statistic of dynamical response
- **Cannot handle nonlinear effects!**

Time domain: **Random Phase Model**

$$x(t) = \sum_{j=1}^{J} b_j \cos(\phi_j - \omega_j t)$$

$$b_j = \sqrt{2S(\omega_j)\Delta\omega}$$

- **Ergodic theorem** relates ensemble statistics to time averages

Challenges - limitations

- Problem: "most" realizations of the statistical steady state don't contain extreme wavegroups, or lead to extreme hull stresses
  - Naive Monte Carlo experiments thus waste a lot of time and effort to precisely learn the effects of quiescent waves!

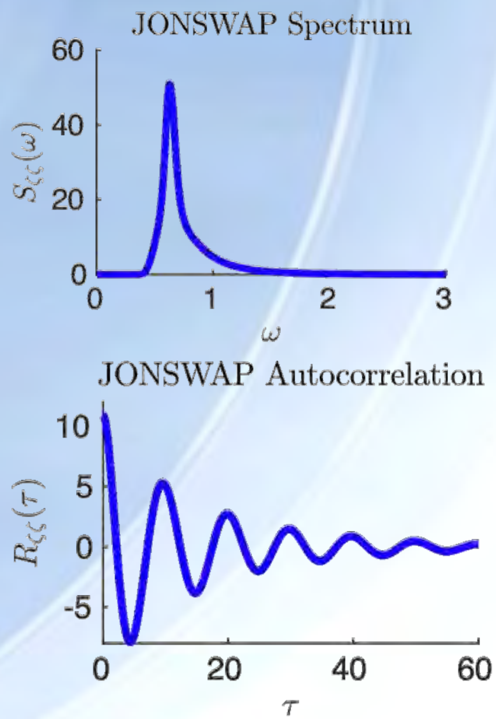
- Alternative: **wave episode** based approach
  - focus simulation/experimental time on **short** extreme wave episodes
  - **reconstruct** the statistical steady state

The plan for the computational method

1. **design wave episode**-based simulation conditions
2. **simulate** mechanical stresses on vessel due to specific sea conditions
3. **construct a surrogate model** to learn the relationship between sea surface elevation (wave episodes) and vessel stresses

- **Active Sampling** to “close the loop” and choose wave episodes

JONSWAP Spectrum



$$S_J(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^r \qquad (1)$$

$$r = \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]$$

$$\alpha = 0.076\left(\frac{U_{10}^2}{Fg}\right)^{0.22} \qquad \gamma = 3.3$$

$$\omega_p = 22\left(\frac{g^2}{U_{10}F}\right)^{\frac{1}{3}} \qquad \sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega > \omega_p \end{cases}$$

- Represents the elevation of surface gravity waves

Karhunen Loève Theorem

- Consider a continuous time random process $x(t)$
  - zero mean, square integrable on $(\Omega, \mathcal{F}, \mathbb{P})$
  - covariance function $R_x(\tau)$

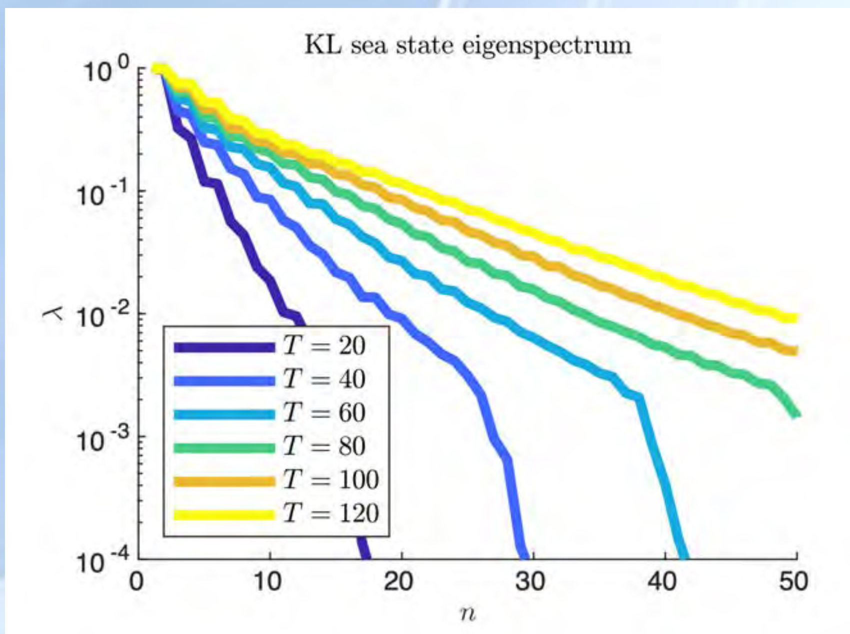
- By Mercer's Theorem, the integral operator

$$T_{R_x}\phi(t) = \int_0^T R_x(t-s)\phi(t)ds, \quad t \in [0,T] \tag{2}$$

  - has an orthonormal basis of eigenvectors $\{e_i(t)\}$ and eigenvalues $\{\lambda_i\}$
  - which we can use to express the random process as

$$\alpha_i = \int_0^T x(t)\hat{e}_{i,T}(t)dt, \quad i = 1, \ldots \qquad x(t) = \sum_{i=1}^{\infty} \alpha_i \hat{e}_{i,T}(t), \quad t \in [0,T]$$

Selection of wave-episode length, T, and number of modes



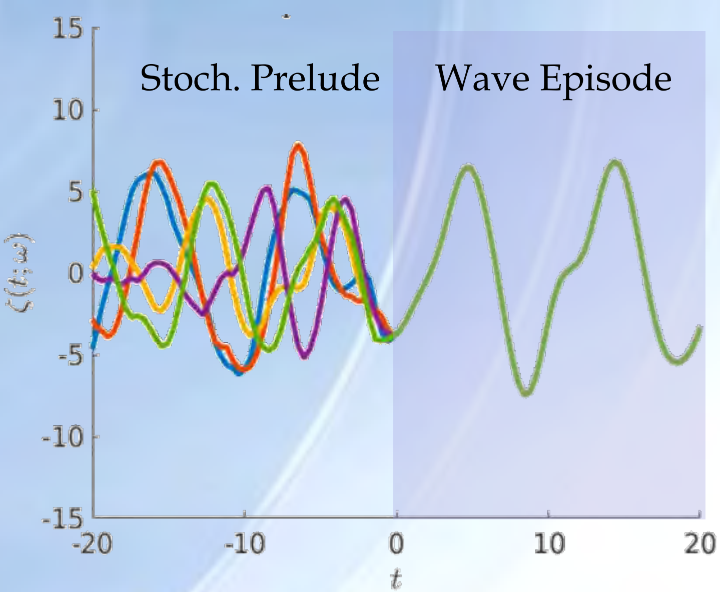
Short T

Poor modeling of transients/memory

Large T

Slow convergence of KL → high dimensionality parameter spaces

Stochastic-prelude for finite-time wave-episodes



Equip each wave-episode with a stochastic prelude that will bring the system close to the stochastic attractor

Definition of SP

Probability measure for wave elevation

$$\mathbb{P}[x(t), t \in \mathbb{R}]$$

Conditional probability measure for wave elevation

$$\mathbb{P}[x(t), t \in \mathbb{R} | \alpha_1, ..., \alpha_n] \triangleq \mathbb{P}\left[x(t), t \in \mathbb{R} \middle| \int_0^T x(s)\hat{e}_{i,T}(s)ds = \begin{cases} \alpha_i, \ i = 1, ..., n \\ 0, i = n+1, ... \end{cases} \right]$$

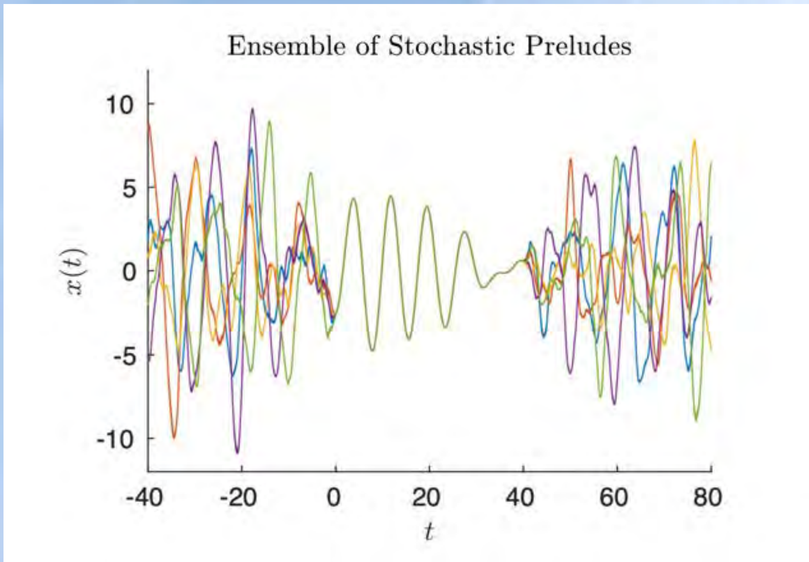
How do we construct numerically the SP for each wave-episode?

Iteratively, using GP conditioning

Using n points in the wave-episode region we find the conditional distribution for the first point of the SP

We sample and then find the conditional pdf for the next point of the SP and repeat

Statistical structure of the Stochastic Prelude



$\boldsymbol{\alpha}-$conditional mean of each wavegroup:

$$\bar{x}(t|\boldsymbol{\alpha}) = \mathbb{E}\left[x(t)|\boldsymbol{\alpha}\right], \quad t \in \mathbb{R}.$$

$$\bar{x}(t|\boldsymbol{\alpha}) = 0, \quad \text{for } t \ll 0 \quad or \quad t \gg T.$$

$\boldsymbol{\alpha}-$conditional variance or stochastic prelude variance:

$$\sigma^2_{SP}(t|\boldsymbol{\alpha}) = \mathbb{E}[(x(t|\boldsymbol{\alpha}) - \mathbb{E}[x(t)|\boldsymbol{\alpha}])^2], \quad t \in \mathbb{R}.$$

→ 0 in the interval [0,T]
→ Gradually increases to σ_x^2 away from [0,T]

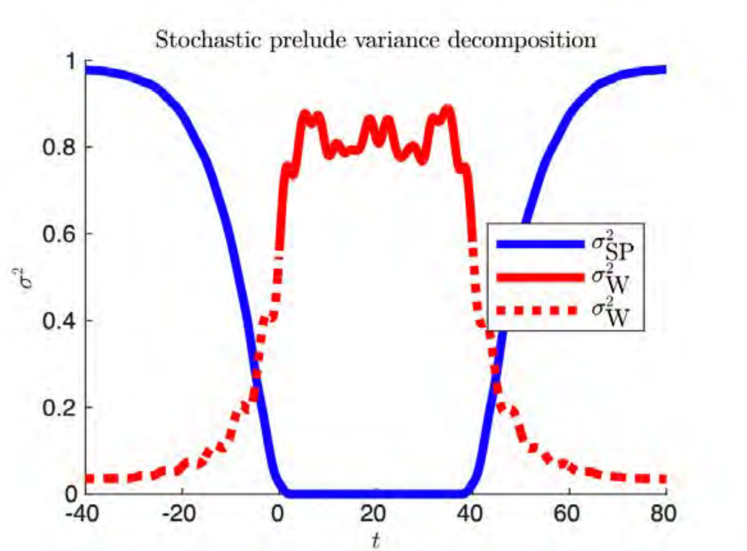
mean of the wavegroups:

$$\mathbb{E}^{\boldsymbol{\alpha}}\left[\bar{x}(t|\boldsymbol{\alpha})\right] = \int_{\mathbb{R}^n} \bar{x}(t|\boldsymbol{\alpha}) p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) d\boldsymbol{\alpha} = 0, \quad t \in \mathbb{R}.$$

variance of the wavegroups (over $\boldsymbol{\alpha}$)

$$\sigma^2_W(t) \triangleq \mathbb{E}^{\boldsymbol{\alpha}}[(\bar{x}(t|\boldsymbol{\alpha}))^2] = \int_{\mathbb{R}^n} \left(\bar{x}(t|\boldsymbol{\alpha})\right)^2 p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) d\boldsymbol{\alpha}, \quad t \in \mathbb{R}.$$

For $t \in [0, T]$ we have a tight bound, $\sigma^2_W(t) < \sigma^2_x$



LAMP: Large Amplitude Motions Program

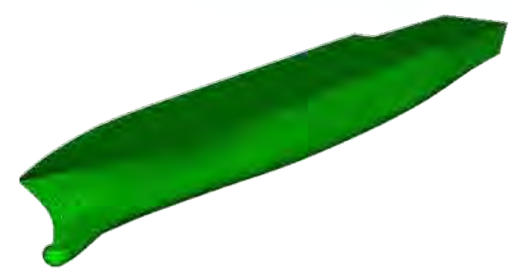
- Large Amplitude Motion Program v4.0.9 (May 2019)
  - Mixed structural and hydrodynamic model of a ship traversing a marine environment
  - Time domain calculations capable of handling **unsteady waves**

+

- Input: Time series for sea surface elevation at global origin ($x = 0$)
- Output: Time series for position, orientation, body forces, internal forces

  θ Pitch

  M_y Vertical Bending Moment (VBM)



LAMP: Large Amplitude Motions Program

Importantly, we treat LAMP as a **Black Box**!

- Each LAMP simulation is "expensive" (3-5 minutes on single core consumer desktop)
- In principle, could be replaced by alternate numerical codes (ComPASS, Das Boot, OpenFOAM, etc) or tow tank physical experiments
- Our goal is to use **minimal** LAMP calls to reconstruct statistical steady state

Stochastic ship responses for each wave-episode

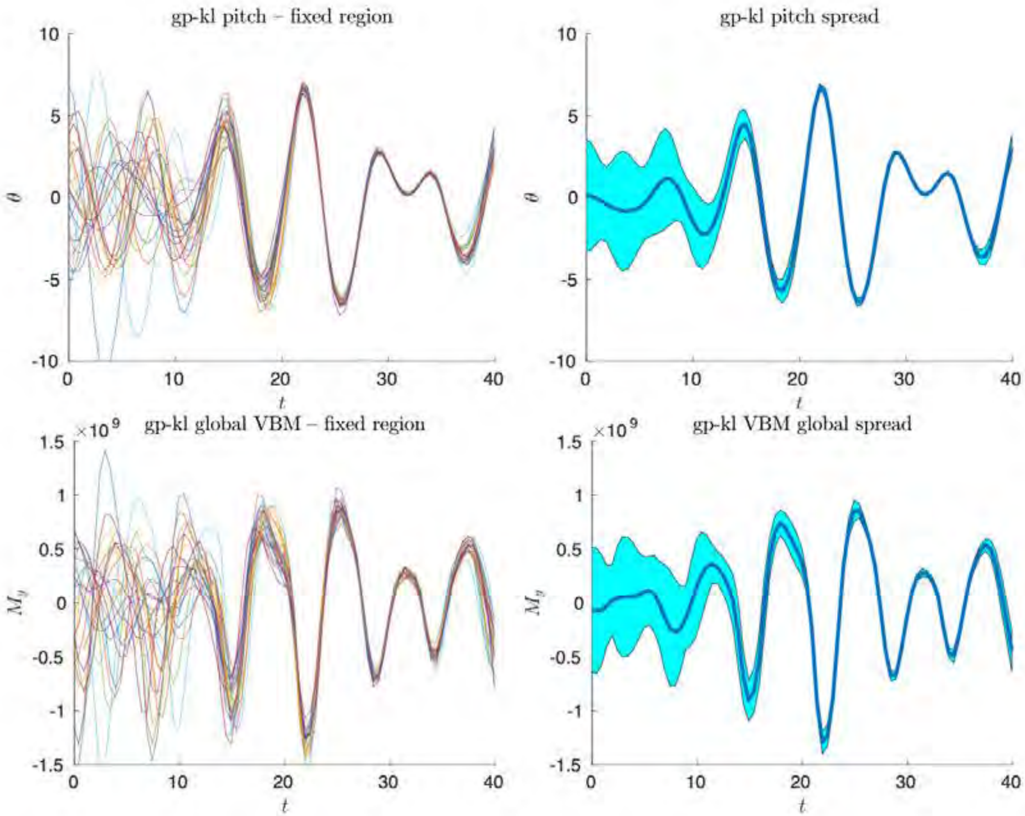
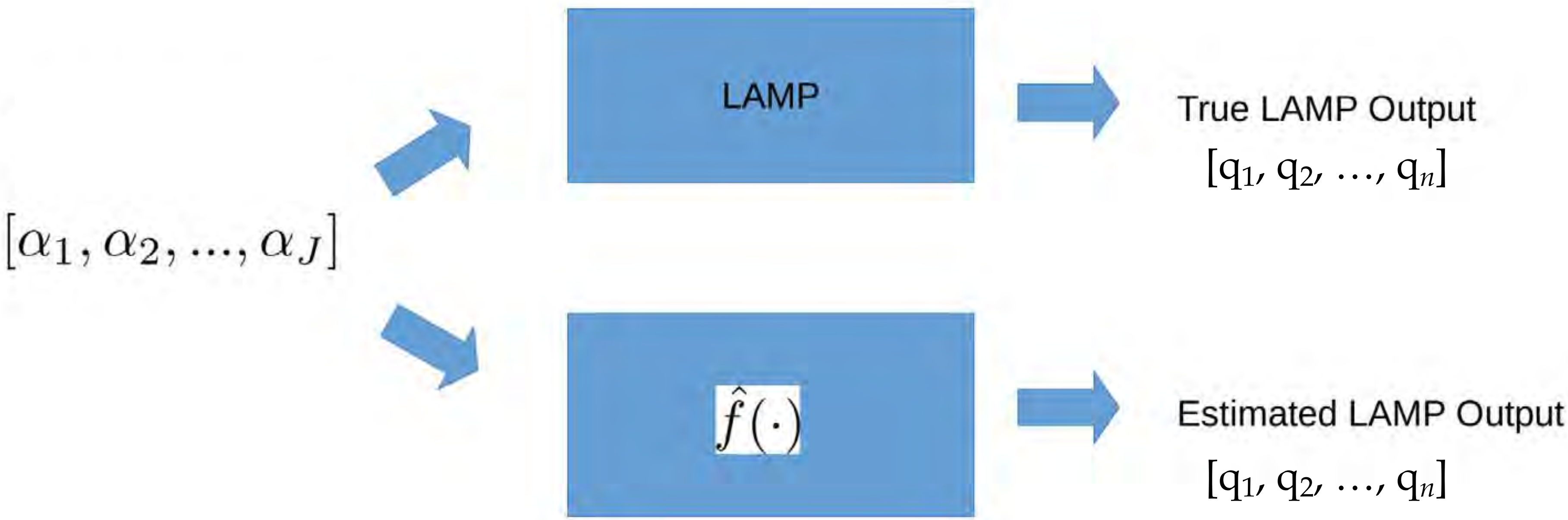


Figure: **Top row**: pitch (θ). **Bottom row**: Vertical Bending Moment (M_y).

- Output time series for the **same** wave episode but **different** stochastic preludes
- Stochastic preludes allows for shorter wave-episodes without loosing information for transients.
- Representation of output time series through a finite-dimensional (random) vector

$$M_y(t|\boldsymbol{\alpha}) = \sum_{i=1}^{n_{out}} q_i(\boldsymbol{\alpha})\hat{\mu}_{i,T}(t), \quad t \in [0,T]$$

Surrogate modeling of ship responses



Surrogate modeling using Gaussian Process Regression

We choose **Gaussian Process Regression** for our surrogate modeling:

- well suited to low dimensional wave episode parametrizations ($n \in [2, 5]$)
- robust to small training sets ($n_s \approx 100$)
- model training is fast ($\approx$ minutes) and model evaluation is **blazing fast**
- built-in estimate of **posterior uncertainty**
- convenient closed forms for **Active Search** math

$$\bar{q}(\alpha) = K_*^\intercal (K + \sigma_n^2 I)^{-1} Q \tag{4}$$

$$\sigma_q^2(\alpha) = K_{**} - K_*^\intercal (K + \sigma_n^2 I)^{-1} K_* \tag{5}$$

Surrogate modeling using Gaussian Process Regression

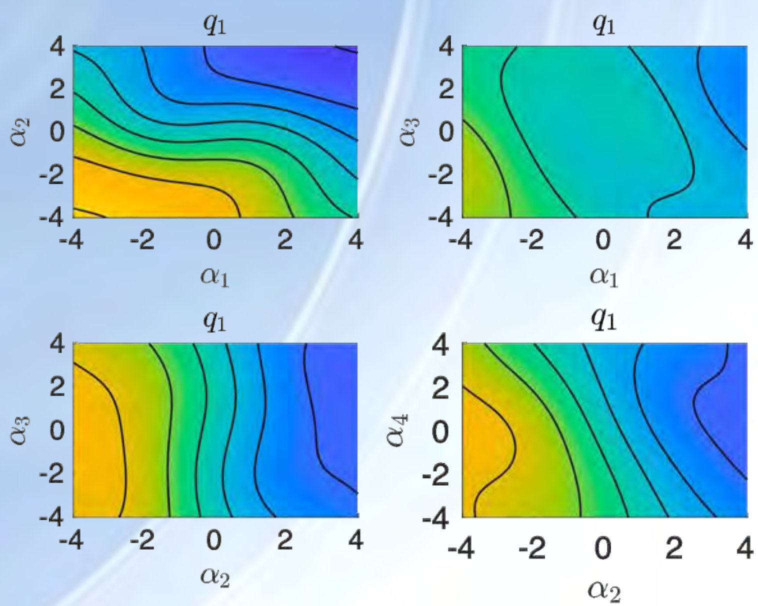


Figure: Sample visualization of surrogate model

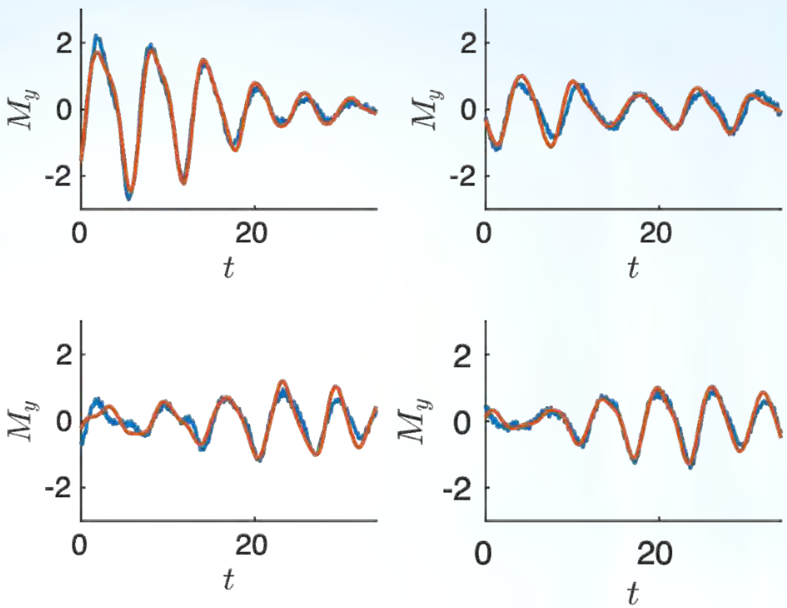


Figure: Sample reconstruction of VBM time series.

Statistical approximation of VBM

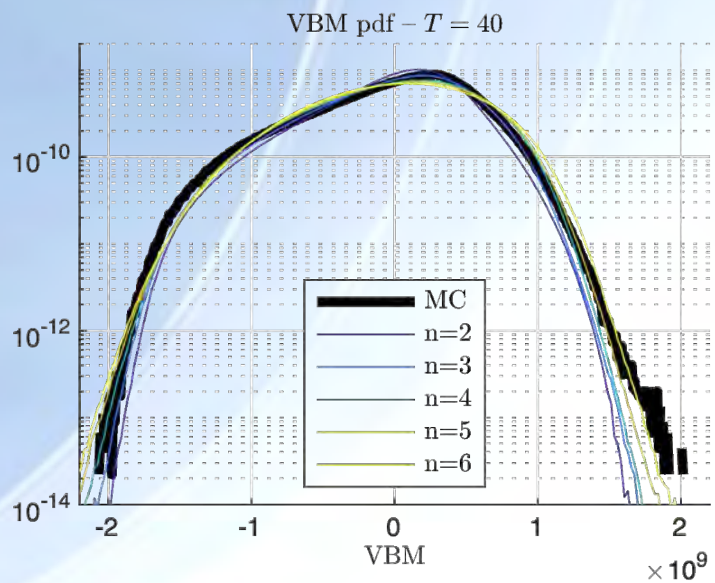


Figure: Sample reconstruction of VBM statistical steady state.

- The MC statistics consist of 3000 hours of steady state simulation, which required **150 CPU days**
  → need to repeat for each new sea state!
- The reconstructed statistics required 625 wave episodes, which required a total of **50 CPU hours**
  → Once simulated, wave episodes are sea state agnostic!
- Time cost for Gaussian process model training and statistical re-sampling are trivial $\sim$ 10 minutes

Transfer learning for different sea states

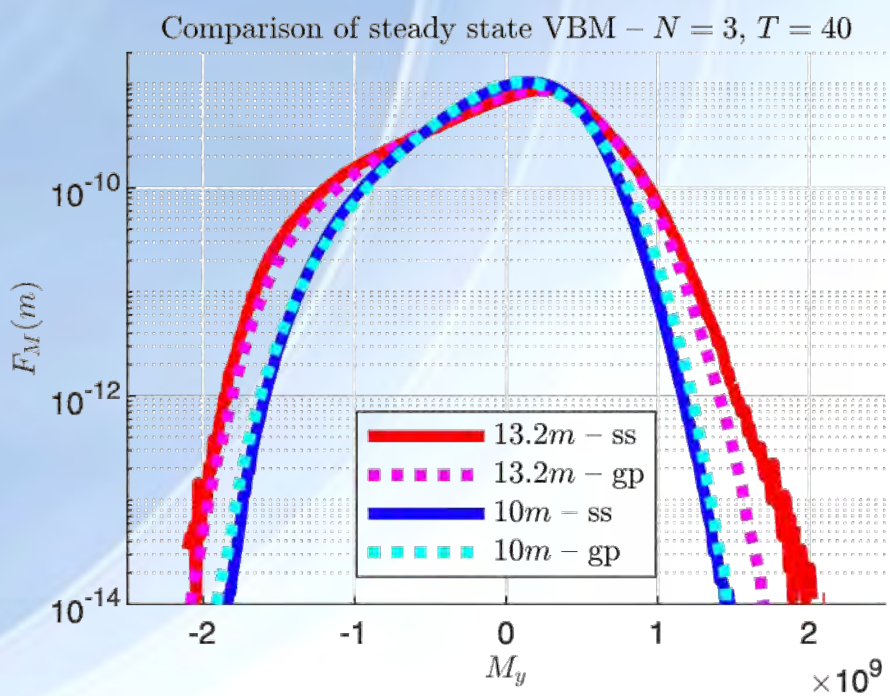


Figure: Spectrum transfer from $H_s = 13.2$ (red/magenta) to $H_s = 10m$ (blue/cyan).

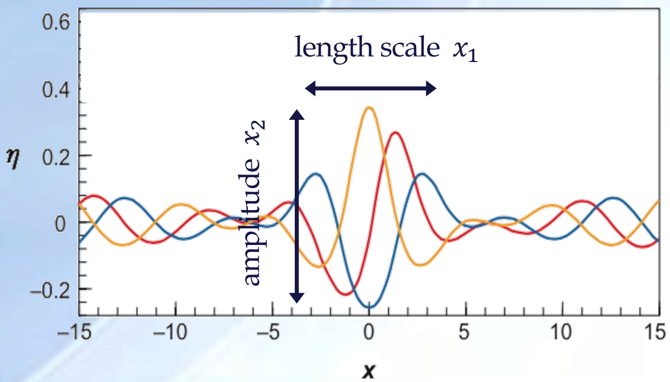
- **Different sea conditions**:
  H_s Significant wave height
- Traditional Monte Carlo simulations must be **repeated each time**
- Wave episode-based data can transferred merely by **adjusting wavegroup probabilities**!

Active sampling to reduce number of simulations

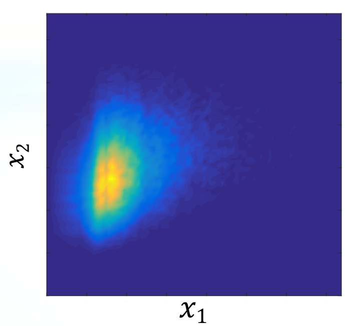
JONSWAP spectral density

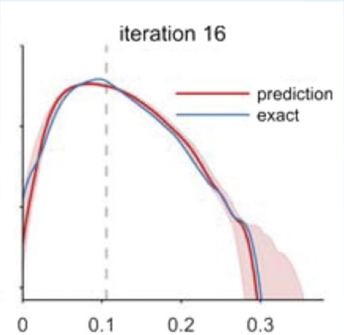
$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-\frac{5}{4}\left(\frac{f_p}{f}\right)^2\right] \cdot \gamma^{\exp\left[\frac{-(f-f_p)^2}{2\delta^2 f_p^2}\right]}$$

2D parametrization of waves



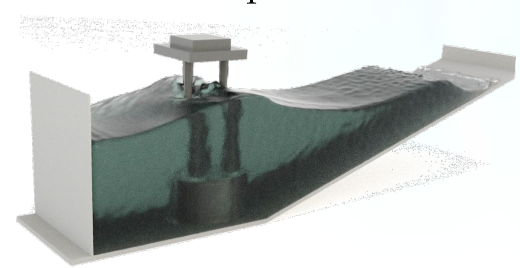
Probability density function of wave parameters





pdf of structural moments with 16 simulations

CFD experiment



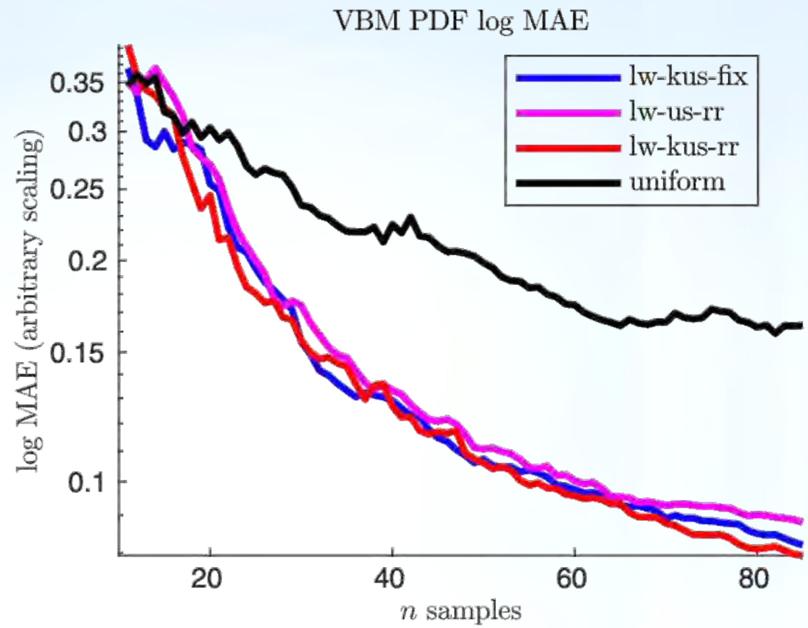
Output pdf acquisition function

$$\min_{x^*} \int \left|\log p_{\bar{y}_N+\sigma_N}(s; x^*) - \log p_{\bar{y}_N}(s)\right| ds$$

[Mohamad & Sapsis, Proc. Nat. Acad. of Sciences, 2018]

Active sampling to reduce number of simulations

- Likelihood Weighted Uncertainty Sampling (LW-US) preferentially learns the **tails** of the distribution
- **Uncertainty Regularization** defeats intrinsic model noise



$$u(\alpha) = \frac{p_A(\alpha)}{p_y(y(\alpha))} \left[\sigma_y^2(\alpha) - \sigma_0^2\right]. \quad (6)$$

Figure: Comparison on learning curve between **uniform sampling** (black) and variations on **Likelihood Weighted Uncertainty Sampling** (colored).

Key points

A) Exactly solvable reduced order models

i) Asymptotic expansion around instantaneous water plane
ii) Monochromatic approximation of waves works well
iii) Each phenomenon has to be accounted carefully
iv) Other issues such as structural response (e.g. whipping) not presented but important

B) Computational method using carefully designed wave-episodes

i) Wave episodes is a promising approach for modeling ship statistics
ii) Important considerations related to wave-episode length and convergence rates
iii) GPR a promising tool for reduced-order modeling but other alternatives are possible
iv) Acceleration of computations by several orders of magnitude
v) Transfer learning for different spectra
vi) Active learning possible to reduce computations even more!

More memories