Effects of Nonlinearity on the Crest shape of Extreme irregular waves

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Overview

Introduction

- A typical depth for the installation of monopile foundations for offshore wind turbines is 10-20m.
- Due to these foundations being typically built as part of offshore wind farms, the determination of crest width of the design wave event is of large importance as to how many of these turbines could be affected by the emergence of such an event.

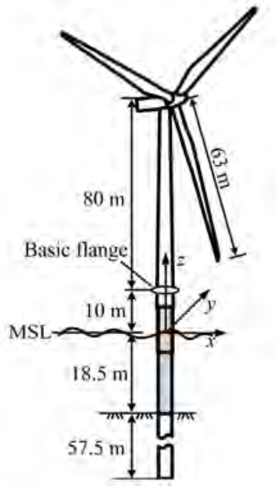


Figure: Monopile foundation (Mo et al., 2017)

Introduction

- Typical breakwaters are also founded at similar water depths.
- The determination of the largest crest elevations of the design wave is very important for the determination of any over-topping and also the calculation of the wave loads acting on the breakwaters and on vessels.
- The affected area hit by an extreme event, associated with the crest width, may have a large effect on the stability and resilience of the structure or ship.

Figure: Breakwater in Volos, Greece

State of the art

Adcock et al. (2015) worked on simulated nonlinear random deep-water directional waves measuring the changes of the crest width during the formulation of large waves compared to linear theory.

→ Numerical calculations were conducted using the modified nonlinear Schrödinger equation, effectively a weakly nonlinear model and a narrow-banded approximation.

→ Even where there is only a marginal change in the maximum surface elevation, compared to linear theory, there is an increase in the crest width and the large waves tend to move to the front of the wave packet; the so-called "walls of water".

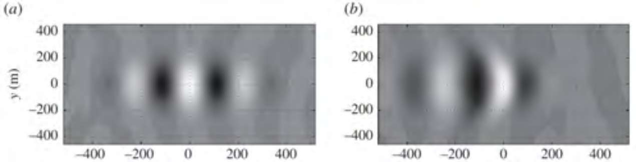


Figure: Average Shape of (a) Linear Waves, (b) Nonlinear Waves (Adcock et al., 2015)

Focus of the present work

- This study focuses on the large events' wave-front of the wave crest, highlighting the necessity to incorporate physics beyond linear theory in relation to the crest width.
- The paper firstly seeks to confirm the deep-water findings of Adcock et al. considering a fully nonlinear model incorporating a broad-banded energy distribution in the various frequencies.
- However, the main purpose of this paper is to investigate whether these findings in deep-water are also relevant to large waves propagating in finite water.
- Such water depths as where the offshore monopile wind farms or breakwaters are founded.
- Simulations are carried for a series of short-crested to long-crested directional focused and random wavefields with a nonlinear numerical model.

HOS-Ocean

- HOS-Ocean, by Ducrozet et al. (2015) is an open-source fully nonlinear model that can simulate the evolution of a fully nonlinear wavefield.
- Based on the High-Order Spectral method, presented in the original work of West et al. and Dommermuth and Yue (1989).
- The present calculations could have also been undertaken with other similar directional wave models, such as BST from Bateman, Swan and Taylor (2001).

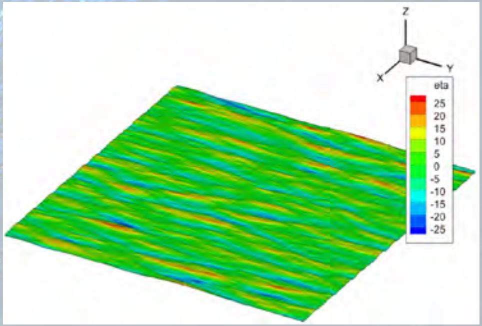


Figure: Random wave simulation using the default Tecplot output of HOS-Ocean

HOS-Ocean

- Working under the potential flow theory, a rectangular fluid domain is considered, with a Cartesian coordinate system. As a result, the continuity equation reduces to the Laplace equation for the velocity potential ϕ (∇ denoting the horizontal gradient operator)

$$\nabla\phi + \frac{\partial^2\phi}{\partial z} = 0 \tag{1}$$

- First, the free surface boundary conditions are defined, described as the free surface elevation η and the free surface velocity potential $\tilde{\phi}$. The free surface boundary conditions read as

$$\frac{\partial\eta}{\partial t} = (1+|\nabla\eta|^2)W - \nabla\tilde{\phi}\cdot\nabla\eta \tag{2}$$

$$\frac{\partial\tilde{\phi}}{\partial t} = -g\eta - \frac{1}{2}|\nabla\tilde{\phi}|^2 + \frac{1}{2}(1+|\nabla\eta|^2)W^2 \tag{3}$$

- where W is the vertical velocity at the free surface which can be evaluated with the HOS scheme of West et al..

HOS-Ocean

- By evaluating the vertical velocity the two unknowns η and $\tilde{\phi}$ can be advanced in time. Periodic lateral boundary conditions are used, assuming a laterally infinite domain. Associating these factors with the Laplace Equation (1) and the bottom boundary condition of zero vertical velocity, the surface properties can be expressed on a spectral basis to allow the use of Fast Fourier Transforms (FFTs).

$$\eta(x,t) = \sum_m B_m^{\eta} exp(ik_m x) \tag{4}$$

$$\tilde{\phi}(x,t) = \sum_m B_m^{\tilde{\phi}} exp(ik_m x) \tag{5}$$

- While knowing the above surface quantities, the HOS scheme referenced above can evaluate the vertical velocity at the free surface W. This relies on a series expansion in wave steepness ε up to the HOS order M with $\phi^{(m)}$ quantities of $\varepsilon^{(m)}$.

$$\phi(x,z,t) = \sum_{m=1}^{M} \phi^{(m)}(x,z,t) \tag{6}$$

HOS-Ocean

- Associating with a Taylor series around $z = 0$ and collecting terms at each order in wave steepness result in a triangular system for $\phi^{(m)}$. This transforms the Dirichlet problem for $\phi(x,z,t)$ into M simpler Dirichlet problems for $\phi^{(m)}(x,0,t)$. Similarly to the velocity potential, a series expansion is applied on the vertical velocity W as seen in Eqn. (7) which leads to its evaluation as seen in Eqn. (8).

$$W^{(m)}(x,t) = \sum_{k=0}^{m-1} \frac{\eta^k}{k!} \frac{\partial^{k+1}\phi^{(m-k)}}{\partial z^{k+1}}(x,0,t) \tag{7}$$

$$W(x,t) = \sum_{m=1}^{M} W^{(m)}(x,z,t) \tag{8}$$

Initial conditions

- The initial conditions for all simulations involved a JONSWAP amplitude spectrum.
- The focused wave events simulated here are based on the celebrated *NewWave* theory and then directionally spread with the directional distribution by Mitsuyasu.

Wavefield / Parameters	short crested	long crested	very long crested
s	7	45	150
N_x	512	1024	
N_y	256	128	
L_x	5000m	5500m	
L_y	3500m		5000m
T_p	10s		
γ	2.5		
Focused $A = \Sigma a_i$	9.5m		
Random $A = \Sigma a_i$	25m		

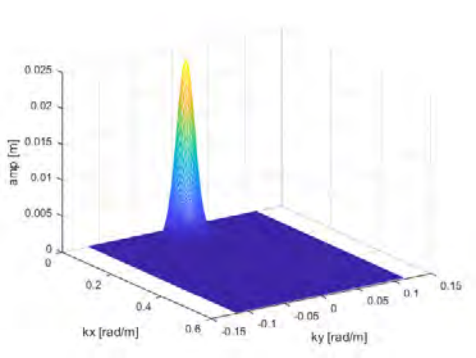


Figure: Input JONSWAP wavenumber spectrum

Initial conditions

- The simulations were first conducted for infinite water depth conditions and then for finite water depth (15m), keeping every other parameter the same. Every run was backward-propagated linearly for 500s or 50 T_p before being run forward nonlinearly.
- To create a random irregular wave train, one has to choose a random number in $[0, 2\pi]$ for the initial phases of the input spectrum.
- Particularly for the formation of a random wavefield, where an extreme event can be formed in the center of the domain, this interval is reduced to $[0, 1.6\pi]$, accumulating part of the energy there.
- This method attempts to simulate the creation of a large event in a random sea-state, while reducing the time-consuming process of identifying extreme events in fully random simulations. The seeding of random numbers in each simulation was kept constant.

Measuring crest width

- In order to measure an effective crest width, a minimum of 30% of the η_{max} of each linear case is considered, so as to measure the portion of the crest that is exceeding this height.
- This is done as a means to evaluate crest width in a repeatable fashion particularly in random simulations, as random phasing can have an effect on the outer edges of crests, by elongating or shortening them where the crest height is insignificant.
- By measuring crest width above a certain threshold, ensures that the measurement accurately represents the effect of the large wave, while taking into account the heights that hold more significance for an event labeled as "extreme".

$\Downarrow$ The results that follow are also compared using contour plots whose view corresponds to a square slice of the domain, with the crest of the extreme event centered. The comparisons are made between the events with the highest crest elevation in each simulation.

Discussing Results: Deep Water

→ Overall, the results in deep water are relatively consistent with the findings of Adcock et al.

- Significant increases in crest width while maintaining a small but significant increase in crest height.
- In focused simulations of the less directional cases the increase is close to 40%.

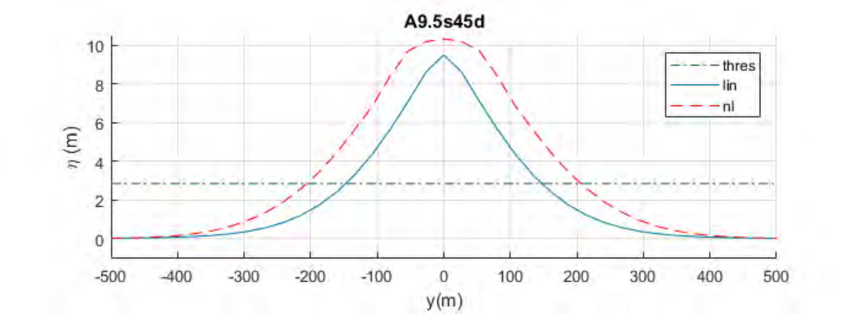


Figure: Comparison between linear and nonlinear crest width in case A9.5s45d (deep)

- The crest has a slight bend around the direction of propagation while having a slimmer profile creating a so-called “wall of water”.

Focused long-crested case (s45), view from the front of wave-group: Deep Water

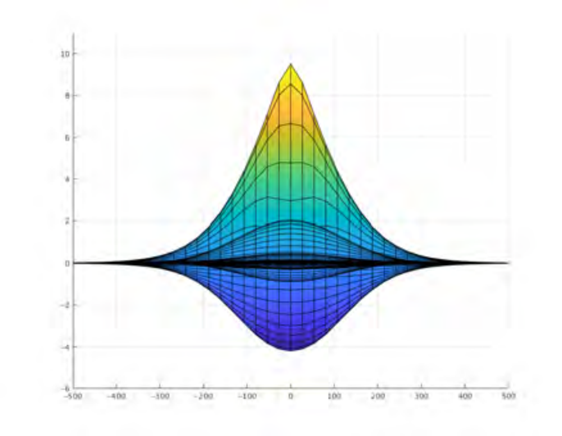


Figure: Linear

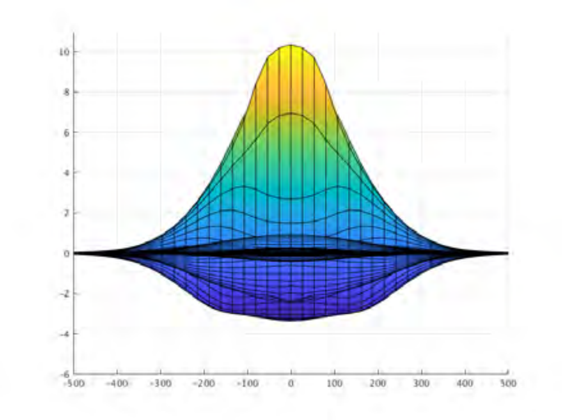


Figure: Nonlinear

Discussing Results: Deep Water

- In random simulations the behavior is quite similar to the focused events but a bit less pronounced.
- Most probably the result of lower steepness compared to the focused cases
- The forced extreme event happens over a randomly phased wavefield, causing an effect of two similarly high crests during linear propagation of case rA25s45d.
- During nonlinear propagation the maximum crest elevation is larger and appears earlier compared to the linear simulation.

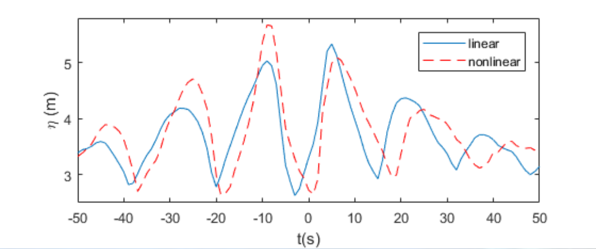
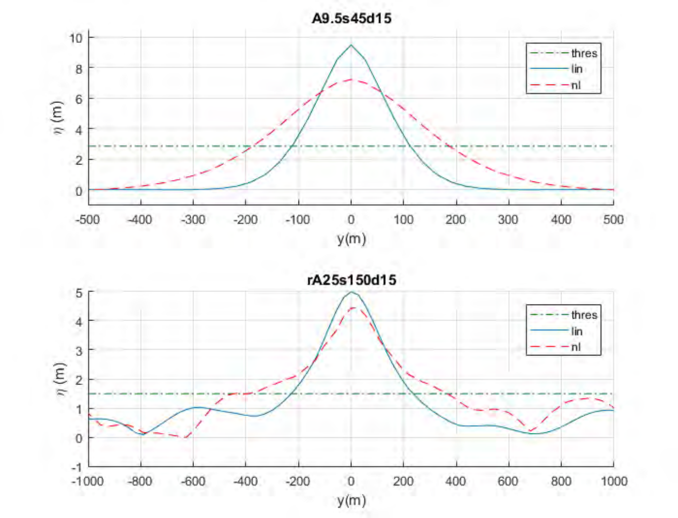


Figure: Time-evolution of maximum surface elevation, η_{max}, in deep water (Case rA25s45d). Comparison between linear and nonlinear simulations

Discussing Results: Intermediate Water

→ In intermediate water, the focused events show a significant difference in the trend shown in deep water.

- Crest height is reduced, but the energy is spread much more widely along the perpendicular direction to propagation (y).
- The disturbance in the wavefield during the extreme events is almost double as wide during nonlinear propagation
- In the very long-crested wavefield (A9.5s150d15) an almost "unidirectional" 1.5km wide wave train is formed



Focused long-crested case (s45), view from in front of wave-group: Intermediate Water

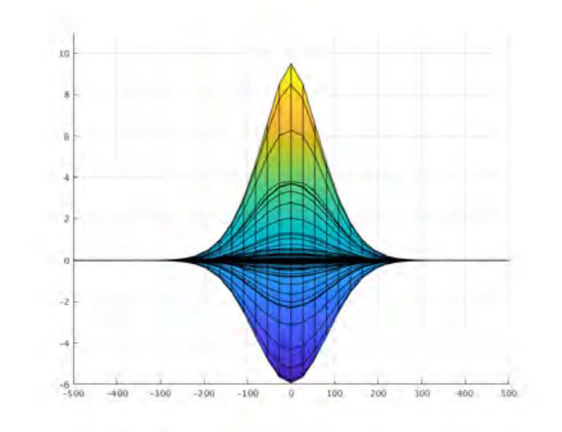


Figure: Linear

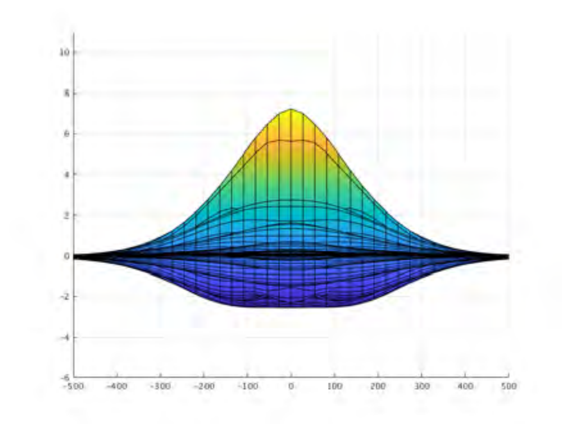


Figure: Nonlinear

Discussing Results: Intermediate Water

- In the very long-crested random case, the behavior is quite similar to the focused event.
- Nonlinearity causing a much wider disturbance in the wavefield than during linear propagation.
- Interestingly, during random simulations the reduction in crest elevation is not as significant, most likely attributed to smaller steepness of the extreme events compared to the focused simulations.

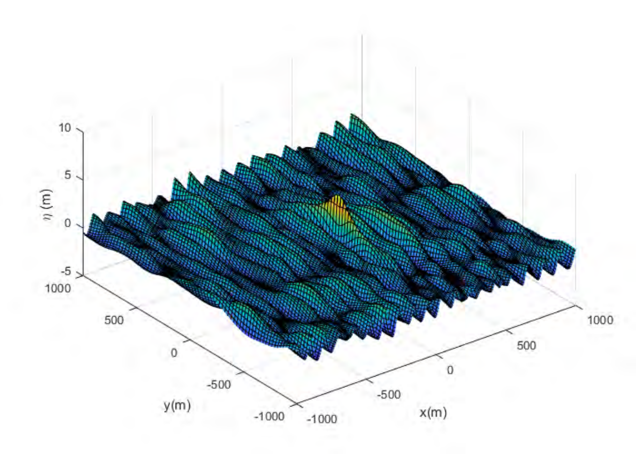


Figure: Surface plot of random event for nonlinear case in intermediate water (rA25s45d15)

Discussing Results: Intermediate Water

- The effect of nonlinearity on random wavefields is also significant.
- Nonlinearity brings forth events which do not resemble the NewWave.

↓ Concerning the short-crested cases, the effects described above are quite less pronounced in both deep and intermediate water.

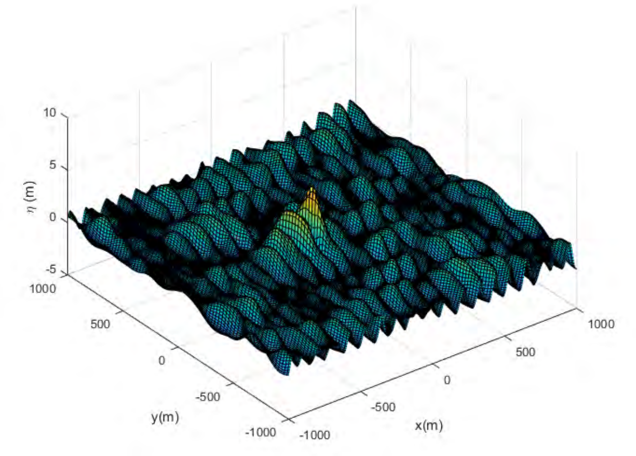


Figure: Surface plot of random event for nonlinear case in intermediate water (rA25s45d15)

Results: Short crested contour plots

Focused short-crested case ($s = 7$) $\Downarrow$

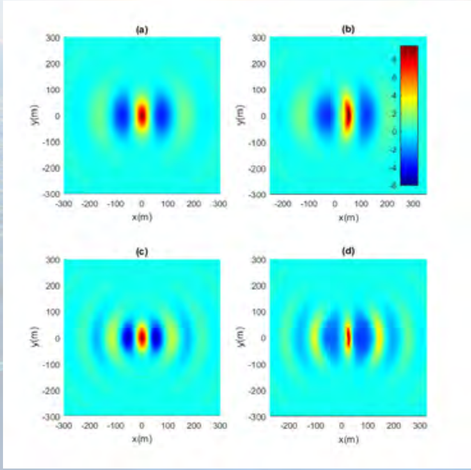


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

$\Downarrow$ Random short-crested case ($s = 7$)

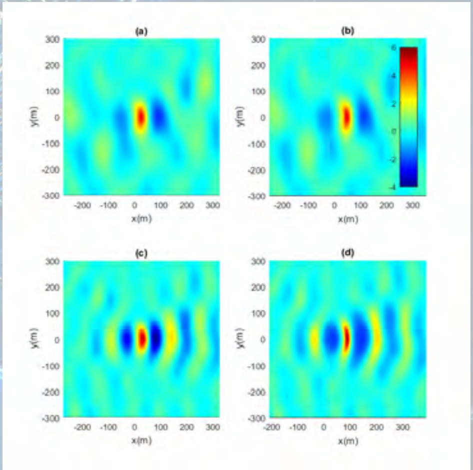


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

Results: Contour plots for focused long-crested events

Focused long-crested case ($s = 45$) $\Downarrow$

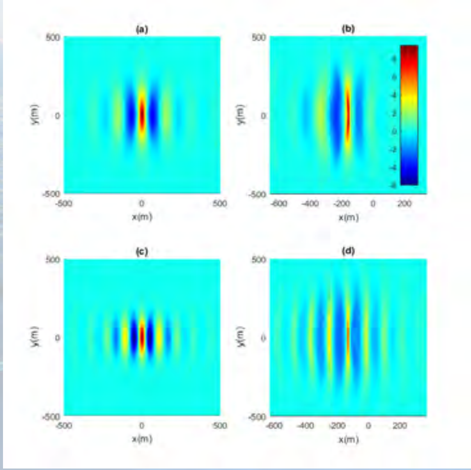


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

$\Downarrow$ Focused very long-crested case ($s = 150$)

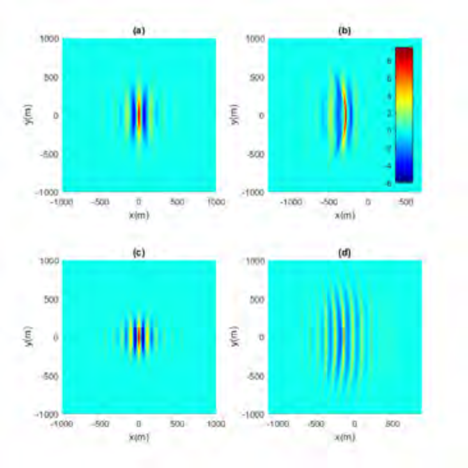


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

Results: Contour plots for random long-crested events

Random long-crested case ($s = 45$) $\Downarrow$

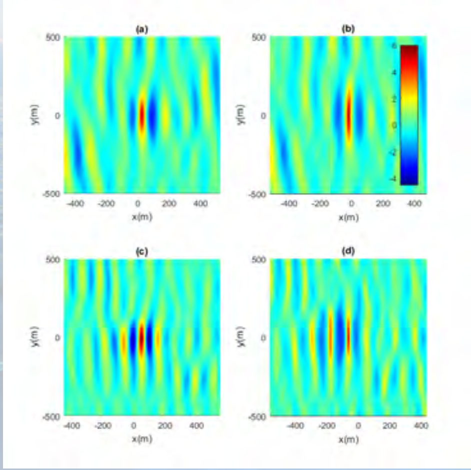


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

$\Downarrow$ Random very long-crested case ($s = 150$)

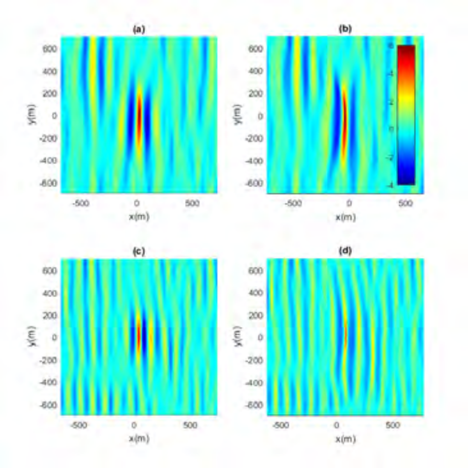


Figure: (a,c) Linear, (b,d) Nonlinear, (a,b) Deep, (c,d) 15m

Discussing crest width

→ Both in deep and intermediate water the increases in crest width are significant.

- Despite the decrease in crest height apparent in intermediate water, nonlinear simulations still present with a very important increase in crest width.
- ⇒ even when compared over the threshold of 30% of the respective linear η_{max}.
- For instance, in the focused case A9.5s150d15, maximum crest height decreases by 36.20% but crest height is over the linear threshold for a 44.08% wider distance.
- In the respective random simulation the behavior is similar ⇒ effective crest width increases by 64.10%.

⟼ This all but confirms the formation of "walls of water" in intermediate depth.

Case	linW (m)	nlW (m)	Difference in %	Threshold (m)
A9.5s7d	125.49	150.48	+19.91%	2.850
A9.5s45d	295.52	412.89	+39.72%	2.850
A9.5s150d	521.29	736.94	+41.37%	2.850
rA25s7d	128.25	136.01	+6.03%	1.453
rA25s45d	318.63	405.76	+27.34%	1.601
rA25s150d	586.78	712.87	+21.49%	1.824
A9.5s7d15	97.58	115.53	+18.39%	2.850
A9.5s45d15	225.69	374.29	+65.84%	2.850
A9.5s150d15	399.58	575.72	+44.08%	2.850
rA25s7d15	109.68	134.70	+22.81%	1.623
rA25s45d15	295.19	312.57	+5.89%	1.745
rA25s150d15	464.82	762.76	+64.10%	1.494

Table: Differences in effective crest width of η_{max} events; Linear (linW) vs Nonlinear (nlW)

Discussing crest width

→ Another way of examining the trend of increasing crest width, is by examining the evolved nonlinear amplitude spectra at η_{max} versus the linear/input spectra.

- In focused cases all evolved nonlinear spectra on the y-axis max-amplitude slice are narrowing compared to linear ⇒ indicating a decrease in directionality.
- The trend seems more pronounced in intermediate water ⇒ consistent with the much wider disturbance caused in the wavefield during focused simulations.

↓ In random simulations, the effect is less discernible due to random phasing, but particularly in intermediate water, in the peak of the spectrum which corresponds to the largest waves, there is significant narrowing.

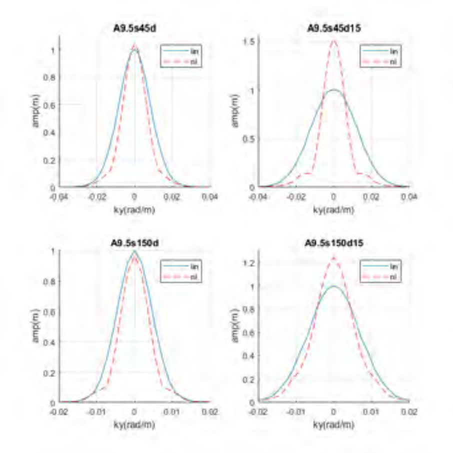


Figure: Comparison between linear and nonlinear evolved spectra for focused cases at time of η_{max}

Crest width $\Longleftrightarrow$ height

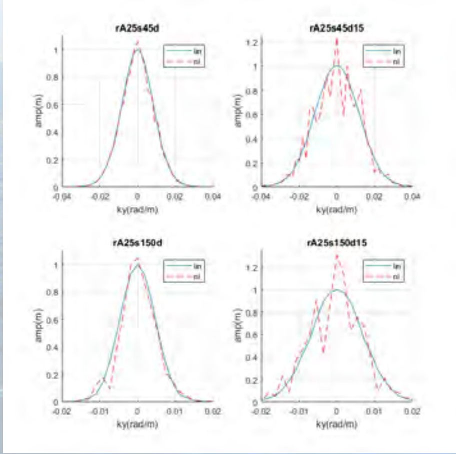


Figure: Comparison between linear and nonlinear evolved spectra for random cases at time of η_{max}

Case	Linear crest η_{max} (m)	Nonlinear crest η_{max} (m)	Difference in %
A9.5s7d	9.500	10.423	+9.72%
A9.5s45d	9.500	10.337	+8.81%
A9.5s150d	9.500	9.720	+2.32%
rA25s7d	4.844	5.327	+9.97%
rA25s45d	5.338	5.678	+6.37%
rA25s150d	6.081	7.084	+16.49%
A9.5s7d15	9.500	9.741	+2.54%
A9.5s45d15	9.500	7.228	-24.02%
A9.5s150d15	9.500	6.061	-36.20%
rA25s7d15	5.410	6.174	+14.12%
rA25s45d15	5.818	5.364	-7.80%
rA25s150d15	4.981	4.426	-11.24%

Table: Differences in maximum crest elevation Linear vs. Nonlinear

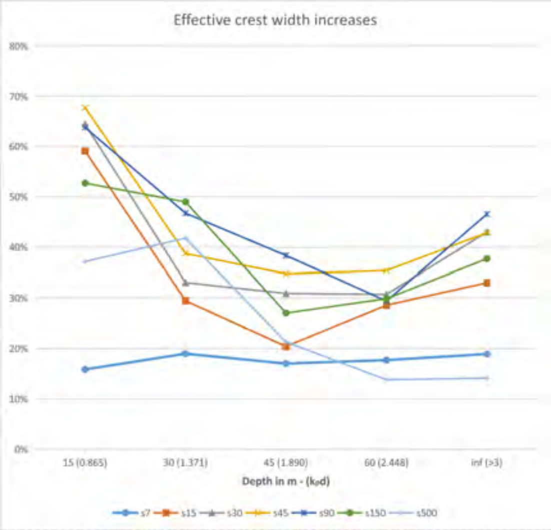
Conclusions

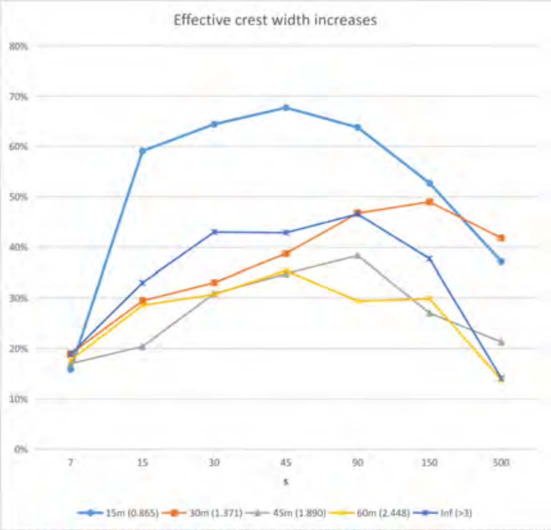
⇑ In all of the above simulations, focused and random, a clear trend of increased crest width during nonlinear formation of large wave events is evident, both in deep and intermediate water.

- While confirming the results of Adcock *et al.* in deep water, this work makes the case for the formation of similar “walls of water” in intermediate depth.
- Despite reduction in crest height compared to linear, nonlinear results show a much wider energy spread along the perpendicular direction to propagation.
- ⇒ particularly in less directional wavefields.
- This has the effect of significantly wider distance over the 30% of the linear η_{max} threshold, despite the reduction in crest height.

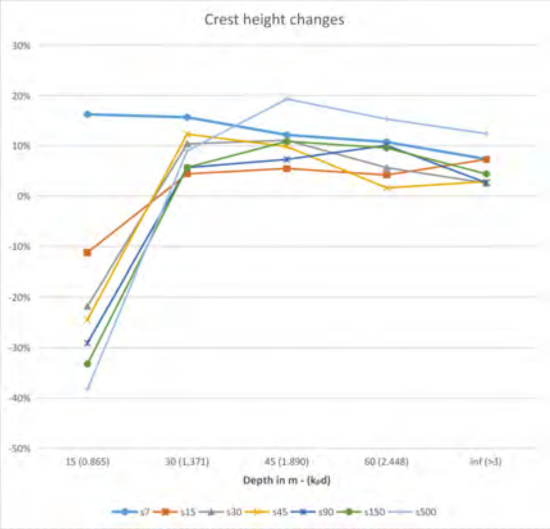
⇓ The aforementioned results warrant further investigation into the effect of the various parameters that could play a role in the general behavior of increased crest width that was presented.

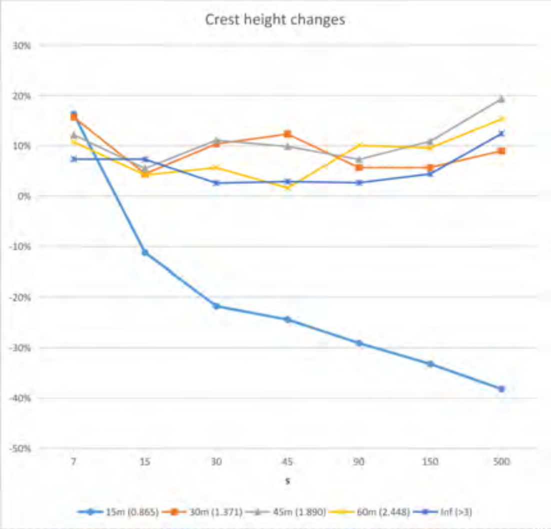
Crest width increases per depth and per directionality





Crest height differences per depth and per directionality





Thank you for your attention!

Questions? Comments?